On isomorphisms and embeddings of $C(K)$ spaces

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Theorem

Under CH, for every K of weight $\leqslant \mathfrak{c}$, $C(K)$ embeds isometrically into $C(\omega^*)$ (which itself is isometric to l_∞/c_0)).

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Todorčević (2011) proved that, consistently, there is a "small" compact K such that $C(K)$ does not embed into $C(\omega^*)$, cf. **Krupski-Marciszewski** (2012).

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For which spaces K there is a totally disconnected L such that $C(K) \sim C(L)$?

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Problem (Argyros & Arvanitakis)

Let K be a compact convex subset of some Banach space which is not metrizable. Can $C(K)$ be isomorphic to $C(L)$, where L is totally disconnected?

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- Suppose that $C(K)$ and $C(L)$ are isomorphic. How K is topologically related to L?
- Suppose that $C(K)$ can be embedded into $C(L)$, where L has some property $\mathcal{P}$. Does K has property $\mathcal{P}$?

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Theorem

Let $T : C(K) \to C(L)$ be a positive isomorphic embedding. Then there is $p \in \mathbb{N}$ and a finite valued mapping $\varphi : L \to [K]^{\leqslant p}$ which is onto ($\bigcup_{y \in L} \varphi(y) = K$) and upper semicontinuous (i.e. $\{y : \varphi(y) \subseteq U\} \subseteq L$ is open for every open $U \subseteq K$).

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If $C(K)$ can be embedded into $C(L)$ by a positive operator then $\tau(K) \leqslant \tau(L)$ and if L is Frechet (or sequentially compact) then K is Frechet (sequentially compact).

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Remark: p is the integer part of $||T|| \cdot ||T^{-1}||$.

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Suppose that there is an operator $T : C(K) \to C(L)$ such that T is either positive isomorphic embedding or an arbitrary isomorphism.
Then there is nonempty open $U \subseteq K$ such that $\overline{U}$ is a continuous image of some compact subspace of L. In fact the family of such U forms a π-base in K.

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Corollary

If $C[0,1]^\kappa \sim C(L)$ then L maps continuously onto $[0,1]^\kappa$.

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Lemma

Let $T : C(K) \to C(L)$ be an embedding such that for $g \in C(K)$

$$m \cdot ||g|| \leqslant ||Tg|| \leqslant ||g||.$$

Then for every $x \in K$ and $m' < m$ there is $y \in L$ such that $\nu_y(\{x\}) > m'$.

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No, under CH (in fact whenever $2^{\omega_1} > \mathfrak{c}$).