Universal sets for σ-ideals

Marcin Michalski, marcin.k.michalski@pwr.edu.pl

Let X be a Polish space and ω^ω denote the Baire space.

Definition 1. *We say that a set $U \subseteq X \times \omega^\omega$ is universal for a family of sets $\mathcal{F} \subseteq P(X)$ if for every $F \in \mathcal{F}$ there exists $y \in \omega^\omega$ such that*

$$U^y = \{x \in X : (x, y) \in U\} = F$$

Well known facts from descriptive set theory say that there exisis universal Borel Σ^0_α set $U \subseteq X \times \omega^\omega$ for Σ^0_α sets, $\alpha < \omega_1$ and analytic universal set for analytic sets.
Let $\mathcal{I}$ be a nontrivial σ-ideal on X possesing a Borel base.

Definition 2. *We say that a set $U \subseteq X \times \omega^\omega$ is universal for the σ-ideal $\mathcal{I}$ if a family of horizontal slices $\{U^y : y \in \omega^\omega\}$ is a Borel base of $\mathcal{I}$.*

We are interested in finding universal sets of possibly low complexity. Our main results include that there are universal sets of minimal complexity (that is, the universal set has the same complexity as the Borel base for a given σ-ideal) for σ-ideal of countable sets, $\mathcal{M}$- meager subsets of X, $\mathcal{N}$- null subsets of the Cantor space 2^ω, $\mathcal{M} \cap \mathcal{N}$ and $\mathcal{E}$- σ-ideal generated by closed null subsets of 2^ω.