Cofinalities of Marczewski-like ideals

Wolfgang Wohofsky

joint work with Jörg Brendle and Yurii Khomskii

Universität Hamburg

wolfgang.wohofsky@gmx.at

Winter School in Abstract Analysis 2017, section Set Theory & Topology
Hejnice, Czech Republic

29th Jan 2017

First a remark concerning the result I presented last year

Theorem (Brendle-W., 2015)

(ZFC) No set of reals of size continuum is "s_0-shiftable".

Definition

A set $Y \subseteq 2^\omega$ is Marczewski null ($Y \in s_0$) $:\Longleftrightarrow$
for any perfect set $P \subseteq 2^\omega$ there is a perfect set $Q \subseteq P$ with $Q \cap Y = \emptyset$.

$$\Longleftrightarrow \forall p \in \mathbb{S} \quad \exists q \leq p \quad [q] \cap Y = \emptyset$$

Definition

A set $X \subseteq 2^\omega$ is s_0-shiftable $:\Longleftrightarrow \forall Y \in s_0 \quad X + Y \neq 2^\omega$
$\Longleftrightarrow \forall Y \in s_0 \quad \exists t \in 2^\omega \quad (X + t) \cap Y = \emptyset$.

Theorem (Brendle-W., 2015, restated more explicitly)

(ZFC) Let $X \subseteq 2^\omega$ with $|X| = \mathfrak{c}$. Then there is a $Y \in s_0$ with $X + Y = 2^\omega$.

. . . -shiftables

$\mathcal{M}$	σ-ideal of meager sets
$\mathcal{N}$	σ-ideal of Lebesgue measure zero ("null") sets
s_0	σ-ideal of Marczewski null sets

$\mathcal{M}$-shiftable	$\iff$	strong measure zero
$\mathcal{N}$-shiftable	$:\iff$	strongly meager
s_0-shiftable		

only the countable sets are $\mathcal{M}$-shiftable	$:\iff$	BC
only the countable sets are $\mathcal{N}$-shiftable	$:\iff$	dBC
only the countable sets are s_0-shiftable	Thilo Weinert $:\iff$	MBC

Consistency of MBC

Theorem (Brendle-W., 2015)

(ZFC) No set of reals of size continuum is "s_0-shiftable".

Theorem (Brendle-W., 2015, restated more explicitly)

(ZFC) Let $X \subseteq 2^\omega$ with $|X| = \mathfrak{c}$. Then there is a $Y \in s_0$ with $X + Y = 2^\omega$.

Corollary

CH implies MBC (i.e., s_0-shiftables $= [2^\omega]^{\leq \aleph_0}$).

The same holds when 2^ω is replaced by any Polish group.

Consistency of MBC

Theorem (Brendle-W., 2015)

(ZFC) No set of reals of size continuum is "s_0-shiftable".

Theorem (Brendle-W., 2015, restated more explicitly)

(ZFC) Let $X \subseteq 2^\omega$ with $|X| = \mathfrak{c}$. Then there is a $Y \in s_0$ with $X + Y = 2^\omega$.

Corollary

CH implies MBC (i.e., s_0-shiftables $= [2^\omega]^{\leq \aleph_0}$).

The same holds when 2^ω is replaced by any Polish group.

Now my actual talk of this year starts.

Cofinalities of Marczewski-like ideals

Wolfgang Wohofsky

joint work with Jörg Brendle and Yurii Khomskii

Universität Hamburg

wolfgang.wohofsky@gmx.at

Winter School in Abstract Analysis 2017, section Set Theory & Topology
Hejnice, Czech Republic

29th Jan 2017

What is a Marczewski-like ideal? We start with a

Definition (Combinatorial tree forcing)

A collection $\mathbb{T}$ of subtrees of $\omega^{<\omega}$ (or $2^{<\omega}$) is a combinatorial tree forcing if

1. $\omega^{<\omega} \in \mathbb{T}$
2. $T \in \mathbb{T} \wedge s \in T \implies T^{[s]} = \{t \in T : t \subseteq s \text{ or } s \subseteq t\} \in \mathbb{T}$
3. large disjoint antichains (in particular implies non-ccc)
   for each $T \in \mathbb{T}$ there is $\{T_\alpha \in \mathbb{T} : \alpha < \mathfrak{c}\}$ such that
   - $T_\alpha \subseteq T$ for each $\alpha < \mathfrak{c}$,
   - $[T_\alpha] \cap [T_\beta] = \emptyset$ for each $\alpha \neq \beta$.
4. (sometimes we also require) homogeneity
5. (we might need a) technical strengthening of large disjoint antichains

$\mathbb{T}$ is ordered by inclusion, i.e., for $S, T \in \mathbb{T}$, $T \leq S$ if $T \subseteq S$.

Examples: Laver/Miller forcing (on $\omega^{<\omega}$), Sacks/Mathias/Silver (on $2^{<\omega}$)

What is a Marczewski-like ideal? We start with a

Definition (Combinatorial tree forcing)

A collection $\mathbb{T}$ of subtrees of $\omega^{<\omega}$ (or $2^{<\omega}$) is a combinatorial tree forcing if

1. $\omega^{<\omega} \in \mathbb{T}$
2. $T \in \mathbb{T} \wedge s \in T \implies T^{[s]} = \{t \in T : t \subseteq s \text{ or } s \subseteq t\} \in \mathbb{T}$
3. large disjoint antichains (in particular implies non-ccc) for each $T \in \mathbb{T}$ there is $\{T_\alpha \in \mathbb{T} : \alpha < \mathfrak{c}\}$ such that
   - $T_\alpha \subseteq T$ for each $\alpha < \mathfrak{c}$,
   - $[T_\alpha] \cap [T_\beta] = \emptyset$ for each $\alpha \neq \beta$.
4. (sometimes we also require) homogeneity
5. (we might need a) technical strengthening of large disjoint antichains

$\mathbb{T}$ is ordered by inclusion, i.e., for $S, T \in \mathbb{T}$, $T \leq S$ if $T \subseteq S$.

Examples: Laver/Miller forcing (on $\omega^{<\omega}$), Sacks/Mathias/Silver (on $2^{<\omega}$)

What is a Marczewski-like ideal? We start with a

Definition (Combinatorial tree forcing)

A collection $\mathbb{T}$ of subtrees of $\omega^{<\omega}$ (or $2^{<\omega}$) is a combinatorial tree forcing if

1. $\omega^{<\omega} \in \mathbb{T}$
2. $T \in \mathbb{T} \wedge s \in T \implies T^{[s]} = \{t \in T : t \subseteq s \text{ or } s \subseteq t\} \in \mathbb{T}$
3. large disjoint antichains (in particular implies non-ccc)
   for each $T \in \mathbb{T}$ there is $\{T_\alpha \in \mathbb{T} : \alpha < \mathfrak{c}\}$ such that
   - $T_\alpha \subseteq T$ for each $\alpha < \mathfrak{c}$,
   - $[T_\alpha] \cap [T_\beta] = \emptyset$ for each $\alpha \neq \beta$.
4. (sometimes we also require) homogeneity
5. (we might need a) technical strengthening of large disjoint antichains

$\mathbb{T}$ is ordered by inclusion, i.e., for $S, T \in \mathbb{T}$, $T \leq S$ if $T \subseteq S$.

Examples: Laver/Miller forcing (on $\omega^{<\omega}$), Sacks/Mathias/Silver (on $2^{<\omega}$)

What is a Marczewski-like ideal? We start with a

Definition (Combinatorial tree forcing)

A collection $\mathbb{T}$ of subtrees of $\omega^{<\omega}$ (or $2^{<\omega}$) is a combinatorial tree forcing if

1. $\omega^{<\omega} \in \mathbb{T}$
2. $T \in \mathbb{T} \wedge s \in T \implies T^{[s]} = \{t \in T : t \subseteq s \text{ or } s \subseteq t\} \in \mathbb{T}$
3. large disjoint antichains (in particular implies non-ccc)
   for each $T \in \mathbb{T}$ there is $\{T_\alpha \in \mathbb{T} : \alpha < \mathfrak{c}\}$ such that
   - $T_\alpha \subseteq T$ for each $\alpha < \mathfrak{c}$,
   - $[T_\alpha] \cap [T_\beta] = \emptyset$ for each $\alpha \neq \beta$.
4. (sometimes we also require) homogeneity
5. (we might need a) technical strengthening of large disjoint antichains

$\mathbb{T}$ is ordered by inclusion, i.e., for $S, T \in \mathbb{T}$, $T \leq S$ if $T \subseteq S$.

Examples: Laver/Miller forcing (on $\omega^{<\omega}$), Sacks/Mathias/Silver (on $2^{<\omega}$)

What is a Marczewski-like ideal? We start with a

Definition (Combinatorial tree forcing)

A collection $\mathbb{T}$ of subtrees of $\omega^{<\omega}$ (or $2^{<\omega}$) is a combinatorial tree forcing if

1. $\omega^{<\omega} \in \mathbb{T}$
2. $T \in \mathbb{T} \wedge s \in T \implies T^{[s]} = \{t \in T : t \subseteq s \text{ or } s \subseteq t\} \in \mathbb{T}$
3. large disjoint antichains (in particular implies non-ccc)
   for each $T \in \mathbb{T}$ there is $\{T_\alpha \in \mathbb{T} : \alpha < \mathfrak{c}\}$ such that
   - $T_\alpha \subseteq T$ for each $\alpha < \mathfrak{c}$,
   - $[T_\alpha] \cap [T_\beta] = \emptyset$ for each $\alpha \neq \beta$.
4. (sometimes we also require) homogeneity
5. (we might need a) technical strengthening of large disjoint antichains

$\mathbb{T}$ is ordered by inclusion, i.e., for $S, T \in \mathbb{T}$, $T \leq S$ if $T \subseteq S$.

Examples: Laver/Miller forcing (on $\omega^{<\omega}$), Sacks/Mathias/Silver (on $2^{<\omega}$)

Let $\mathbb{T}$ be a combinatorial tree forcing, and let $X \subseteq \omega^\omega$ (or $X \subseteq 2^\omega$).

Definition (Marczewski-like ideal t^0 associated to $\mathbb{T}$)

$X \in t^0 \quad :\Longleftrightarrow \quad \forall S \in \mathbb{T} \quad \exists T \leq S \quad [T] \cap X = \emptyset.$

(More or less well-known) examples:

- Marczewski ideal s^0 (associated to Sacks forcing $\mathbb{S}$)
- ideal r^0 of nowhere Ramsey sets (associated to Mathias forcing $\mathbb{R}$)
- ideal v^0 (associated to Silver forcing $\mathbb{V}$)
- ideal ℓ^0 (associated to Laver forcing $\mathbb{L}$)
- ideal m^0 (associated to Miller forcing $\mathbb{M}$)

Definition (Cofinality of an ideal $\mathcal{I}$)

The cofinality $\mathrm{cof}(\mathcal{I})$ is the smallest cardinality of a basis $\mathcal{J}$ of $\mathcal{I}$, i.e., a family $\mathcal{J} \subseteq \mathcal{I}$ such that every member of $\mathcal{I}$ is contained in a member of $\mathcal{J}$.

Let $\mathbb{T}$ be a combinatorial tree forcing, and let $X \subseteq \omega^\omega$ (or $X \subseteq 2^\omega$).

Definition (Marczewski-like ideal t^0 associated to $\mathbb{T}$)

$X \in t^0 \quad :\Longleftrightarrow \quad \forall S \in \mathbb{T} \quad \exists T \leq S \quad [T] \cap X = \emptyset.$

(More or less well-known) examples:

- Marczewski ideal s^0 (associated to Sacks forcing $\mathbb{S}$)
- ideal r^0 of nowhere Ramsey sets (associated to Mathias forcing $\mathbb{R}$)
- ideal v^0 (associated to Silver forcing $\mathbb{V}$)
- ideal ℓ^0 (associated to Laver forcing $\mathbb{L}$)
- ideal m^0 (associated to Miller forcing $\mathbb{M}$)

Definition (Cofinality of an ideal $\mathcal{I}$)

The cofinality $\mathrm{cof}(\mathcal{I})$ is the smallest cardinality of a basis $\mathcal{J}$ of $\mathcal{I}$, i.e., a family $\mathcal{J} \subseteq \mathcal{I}$ such that every member of $\mathcal{I}$ is contained in a member of $\mathcal{J}$.

Let $\mathbb{T}$ be a combinatorial tree forcing, and let $X \subseteq \omega^\omega$ (or $X \subseteq 2^\omega$).

Definition (Marczewski-like ideal t^0 associated to $\mathbb{T}$)

$X \in t^0 \quad :\Longleftrightarrow \quad \forall S \in \mathbb{T} \quad \exists T \leq S \quad [T] \cap X = \emptyset.$

(More or less well-known) examples:

- Marczewski ideal s^0 (associated to Sacks forcing $\mathbb{S}$)
- ideal r^0 of nowhere Ramsey sets (associated to Mathias forcing $\mathbb{R}$)
- ideal v^0 (associated to Silver forcing $\mathbb{V}$)
- ideal ℓ^0 (associated to Laver forcing $\mathbb{L}$)
- ideal m^0 (associated to Miller forcing $\mathbb{M}$)

Definition (Cofinality of an ideal $\mathcal{I}$)

The cofinality $\mathrm{cof}(\mathcal{I})$ is the smallest cardinality of a basis $\mathcal{J}$ of $\mathcal{I}$, i.e., a family $\mathcal{J} \subseteq \mathcal{I}$ such that every member of $\mathcal{I}$ is contained in a member of $\mathcal{J}$.

$$\mathrm{cof}(t^0) = ?$$

- $\mathrm{add}(t^0)$
- $\mathrm{cov}(t^0)$
- $\mathrm{non}(t^0)$
- $\mathrm{cof}(t^0)$

Large disjoint antichains $\longrightarrow$ $\mathrm{non}(t^0) = \mathfrak{c}$;

$\mathrm{cof}(\mathcal{I}) \geq \mathrm{non}(\mathcal{I})$ for any non-trivial ideal $\mathcal{I}$;

hence, $\mathrm{cof}(t^0) \geq \mathfrak{c}$.

¿ $\mathrm{cof}(t^0) = \mathfrak{c}$ or $\mathrm{cof}(t^0) > c$?

$\mathrm{cof}(t^0) = ?$

- $\mathrm{add}(t^0)$
- $\mathrm{cov}(t^0)$
- $\mathrm{non}(t^0)$
- $\mathrm{cof}(t^0)$

Large disjoint antichains $\longrightarrow$ $\mathrm{non}(t^0) = \mathfrak{c}$;

$\mathrm{cof}(\mathcal{I}) \geq \mathrm{non}(\mathcal{I})$ for any non-trivial ideal $\mathcal{I}$;

hence, $\mathrm{cof}(t^0) \geq \mathfrak{c}$.

$$¿\ \mathrm{cof}(t^0) = \mathfrak{c} \quad \text{or} \quad \mathrm{cof}(t^0) > c\ ?$$

$\mathrm{cof}(t^0) = ?$

- $\mathrm{add}(t^0)$
- $\mathrm{cov}(t^0)$
- $\mathrm{non}(t^0)$
- $\mathrm{cof}(t^0)$

Large disjoint antichains $\longrightarrow$ $\mathrm{non}(t^0) = \mathfrak{c}$;

$\mathrm{cof}(\mathcal{I}) \geq \mathrm{non}(\mathcal{I})$ for any non-trivial ideal $\mathcal{I}$;

hence, $\mathrm{cof}(t^0) \geq \mathfrak{c}$.

¿ $\mathrm{cof}(t^0) = \mathfrak{c}$ or $\mathrm{cof}(t^0) > c$?

$\mathrm{cof}(t^0) = ?$

- $\mathrm{add}(t^0)$
- $\mathrm{cov}(t^0)$
- $\mathrm{non}(t^0)$
- $\mathrm{cof}(t^0)$

Large disjoint antichains $\longrightarrow$ $\mathrm{non}(t^0) = \mathfrak{c}$;

$\mathrm{cof}(\mathcal{I}) \geq \mathrm{non}(\mathcal{I})$ for any non-trivial ideal $\mathcal{I}$;

hence, $\mathrm{cof}(t^0) \geq \mathfrak{c}$.

¿ $\mathrm{cof}(t^0) = \mathfrak{c}$ or $\mathrm{cof}(t^0) > c$?

$\mathrm{cof}(t^0) = ?$

- $\mathrm{add}(t^0)$
- $\mathsf{cov}(t^0)$
- $\mathsf{non}(t^0)$
- $\mathrm{cof}(t^0)$

Large disjoint antichains $\longrightarrow$ $\mathsf{non}(t^0) = \mathfrak{c}$;

$\mathrm{cof}(\mathcal{I}) \geq \mathsf{non}(\mathcal{I})$ for any non-trivial ideal $\mathcal{I}$;

hence, $\mathrm{cof}(t^0) \geq \mathfrak{c}$.

¿ $\mathrm{cof}(t^0) = \mathfrak{c}$ or $\mathrm{cof}(t^0) > c$?

$\mathrm{cof}(t^0) = ?$

- $\mathrm{add}(t^0)$
- $\mathsf{cov}(t^0)$
- $\mathsf{non}(t^0)$
- $\mathrm{cof}(t^0)$

Large disjoint antichains $\longrightarrow$ $\mathsf{non}(t^0) = \mathfrak{c}$;

$\mathrm{cof}(\mathcal{I}) \geq \mathsf{non}(\mathcal{I})$ for any non-trivial ideal $\mathcal{I}$;

hence, $\mathrm{cof}(t^0) \geq \mathfrak{c}$.

¿ $\mathrm{cof}(t^0) = \mathfrak{c}$ or $\mathrm{cof}(t^0) > c$?

Definition

$\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

Definition

$\mathbb{T}$ has the incompatibility shrinking property if for any $T \in \mathbb{T}$ and any family $(S_\alpha : \alpha < \mu)$ of size $\mu < \mathfrak{c}$ with S_α incompatible with T for all $\alpha < \mu$, one can find $T' \leq T$ such that $[T']$ is disjoint from all the $[S_\alpha]$.

Proposition

$\mathbb{T}$ incompatibility shrinking prop $\Longrightarrow$ $\mathbb{T}$ disjoint maximal antichain prop

$$\mathbb{T} \text{ disjoint maximal antichain prop } \Longrightarrow cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Several forcings have the incompatibility shrinking prop. provably in ZFC:

	Sacks forcing $\mathbb{S}$	Mathias forcing $\mathbb{R}$	Silver forcing $\mathbb{V}$
So, $\mathrm{ZFC} \vdash$	$cf(\mathrm{cof}(s^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(r^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(v^0)) > \mathfrak{c}$

Definition

$\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

Definition

$\mathbb{T}$ has the incompatibility shrinking property if for any $T \in \mathbb{T}$ and any family $(S_\alpha : \alpha < \mu)$ of size $\mu < \mathfrak{c}$ with S_α incompatible with T for all $\alpha < \mu$, one can find $T' \leq T$ such that $[T']$ is disjoint from all the $[S_\alpha]$.

Proposition

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$$\mathbb{T} \text{ disjoint maximal antichain prop} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Several forcings have the incompatibility shrinking prop. provably in ZFC:

	Sacks forcing $\mathbb{S}$	Mathias forcing $\mathbb{R}$	Silver forcing $\mathbb{V}$
So, ZFC $\vdash$	$cf(\mathrm{cof}(s^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(r^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(v^0)) > \mathfrak{c}$

Definition

$\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

Definition

$\mathbb{T}$ has the incompatibility shrinking property if for any $T \in \mathbb{T}$ and any family $(S_\alpha : \alpha < \mu)$ of size $\mu < \mathfrak{c}$ with S_α incompatible with T for all $\alpha < \mu$, one can find $T' \leq T$ such that $[T']$ is disjoint from all the $[S_\alpha]$.

Proposition

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$\mathbb{T}$ disjoint maximal antichain prop $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$

Several forcings have the incompatibility shrinking prop. provably in ZFC:

	Sacks forcing $\mathbb{S}$	Mathias forcing $\mathbb{R}$	Silver forcing $\mathbb{V}$
So, $\mathrm{ZFC} \vdash$	$cf(\mathrm{cof}(s^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(r^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(v^0)) > \mathfrak{c}$

Definition

$\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

Definition

$\mathbb{T}$ has the incompatibility shrinking property if for any $T \in \mathbb{T}$ and any family $(S_\alpha : \alpha < \mu)$ of size $\mu < \mathfrak{c}$ with S_α incompatible with T for all $\alpha < \mu$, one can find $T' \leq T$ such that $[T']$ is disjoint from all the $[S_\alpha]$.

Proposition

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$$\mathbb{T} \text{ disjoint maximal antichain prop} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Several forcings have the incompatibility shrinking prop. provably in ZFC:

	Sacks forcing $\mathbb{S}$	Mathias forcing $\mathbb{R}$	Silver forcing $\mathbb{V}$
So, $\mathrm{ZFC} \vdash$	$cf(\mathrm{cof}(s^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(r^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(v^0)) > \mathfrak{c}$

Definition

$\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

Definition

$\mathbb{T}$ has the incompatibility shrinking property if for any $T \in \mathbb{T}$ and any family $(S_\alpha : \alpha < \mu)$ of size $\mu < \mathfrak{c}$ with S_α incompatible with T for all $\alpha < \mu$, one can find $T' \leq T$ such that $[T']$ is disjoint from all the $[S_\alpha]$.

Proposition

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$$\mathbb{T} \text{ disjoint maximal antichain prop} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Several forcings have the incompatibility shrinking prop. provably in ZFC:

	Sacks forcing $\mathbb{S}$	Mathias forcing $\mathbb{R}$	Silver forcing $\mathbb{V}$
So, ZFC $\vdash$	$cf(\mathrm{cof}(s^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(r^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(v^0)) > \mathfrak{c}$

Definition

$\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

Definition

$\mathbb{T}$ has the incompatibility shrinking property if for any $T \in \mathbb{T}$ and any family $(S_\alpha : \alpha < \mu)$ of size $\mu < \mathfrak{c}$ with S_α incompatible with T for all $\alpha < \mu$, one can find $T' \leq T$ such that $[T']$ is disjoint from all the $[S_\alpha]$.

Proposition

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$$\mathbb{T} \text{ disjoint maximal antichain prop} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Several forcings have the incompatibility shrinking prop. provably in ZFC:

	Sacks forcing $\mathbb{S}$	Mathias forcing $\mathbb{R}$	Silver forcing $\mathbb{V}$
So, ZFC $\vdash$	$cf(\mathrm{cof}(s^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(r^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(v^0)) > \mathfrak{c}$

Definition

$\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

Definition

$\mathbb{T}$ has the incompatibility shrinking property if for any $T \in \mathbb{T}$ and any family $(S_\alpha : \alpha < \mu)$ of size $\mu < \mathfrak{c}$ with S_α incompatible with T for all $\alpha < \mu$, one can find $T' \leq T$ such that $[T']$ is disjoint from all the $[S_\alpha]$.

Proposition

$\mathbb{T}$ incompatibility shrinking prop $\Longrightarrow$ $\mathbb{T}$ disjoint maximal antichain prop

$$\mathbb{T} \text{ disjoint maximal antichain prop} \Longrightarrow cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Several forcings have the incompatibility shrinking prop. provably in ZFC:

Sacks forcing $\mathbb{S}$ Mathias forcing $\mathbb{R}$ Silver forcing $\mathbb{V}$

So, ZFC $\vdash$ $cf(\mathrm{cof}(s^0)) > \mathfrak{c}$ $cf(\mathrm{cof}(r^0)) > \mathfrak{c}$ $cf(\mathrm{cof}(v^0)) > \mathfrak{c}$

Definition

$\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

Definition

$\mathbb{T}$ has the incompatibility shrinking property if for any $T \in \mathbb{T}$ and any family $(S_\alpha : \alpha < \mu)$ of size $\mu < \mathfrak{c}$ with S_α incompatible with T for all $\alpha < \mu$, one can find $T' \leq T$ such that $[T']$ is disjoint from all the $[S_\alpha]$.

Proposition

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$$\mathbb{T} \text{ disjoint maximal antichain prop} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Several forcings have the incompatibility shrinking prop. provably in ZFC:

Sacks forcing $\mathbb{S}$ Mathias forcing $\mathbb{R}$ Silver forcing $\mathbb{V}$

So, ZFC $\vdash cf(\mathrm{cof}(s^0)) > \mathfrak{c}$ $cf(\mathrm{cof}(r^0)) > \mathfrak{c}$ $cf(\mathrm{cof}(v^0)) > \mathfrak{c}$

Definition

$\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

Definition

$\mathbb{T}$ has the incompatibility shrinking property if for any $T \in \mathbb{T}$ and any family $(S_\alpha : \alpha < \mu)$ of size $\mu < \mathfrak{c}$ with S_α incompatible with T for all $\alpha < \mu$, one can find $T' \leq T$ such that $[T']$ is disjoint from all the $[S_\alpha]$.

Proposition

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$$\mathbb{T} \text{ disjoint maximal antichain prop} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Several forcings have the incompatibility shrinking prop. provably in ZFC:

	Sacks forcing $\mathbb{S}$	Mathias forcing $\mathbb{R}$	Silver forcing $\mathbb{V}$
So, $\mathrm{ZFC} \vdash$	$cf(\mathrm{cof}(s^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(r^0)) > \mathfrak{c}$	$cf(\mathrm{cof}(v^0)) > \mathfrak{c}$

Proposition (from previous slide)

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$\mathbb{T}$ disjoint maximal antichain prop $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$

Assume there is a fusion argument for $\mathbb{T}$ (in this case, t^0 is a σ-ideal).

Then: CH $\implies$ $\mathbb{T}$ has the incompatibility shrinking property
So: CH $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$

For Laver and Miller forcing, weaker hypotheses are sufficient:

Proposition

$\mathfrak{b} = \mathfrak{c} \implies$ Laver forcing $\mathbb{L}$ has the incompatibility shrinking property
$\mathfrak{d} = \mathfrak{c} \implies$ Miller forcing $\mathbb{M}$ has the incompatibility shrinking property

Question

Does $\mathbb{L}$ (or $\mathbb{M}$) have the disjoint maximal antichain property in ZFC?

Proposition (from previous slide)

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$\mathbb{T}$ disjoint maximal antichain prop $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$

Assume there is a fusion argument for $\mathbb{T}$ (in this case, t^0 is a σ-ideal).

Then: CH $\implies$ $\mathbb{T}$ has the incompatibility shrinking property

So: CH $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$

For Laver and Miller forcing, weaker hypotheses are sufficient:

Proposition

$\mathfrak{b} = \mathfrak{c} \implies$ Laver forcing $\mathbb{L}$ has the incompatibility shrinking property

$\mathfrak{d} = \mathfrak{c} \implies$ Miller forcing $\mathbb{M}$ has the incompatibility shrinking property

Question

Does $\mathbb{L}$ (or $\mathbb{M}$) have the disjoint maximal antichain property in ZFC?

Proposition (from previous slide)

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$\mathbb{T}$ disjoint maximal antichain prop $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$

Assume there is a fusion argument for $\mathbb{T}$ (in this case, t^0 is a σ-ideal).

Then: CH $\implies$ $\mathbb{T}$ has the incompatibility shrinking property

So: CH $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$

For Laver and Miller forcing, weaker hypotheses are sufficient:

Proposition

$\mathfrak{b} = \mathfrak{c} \implies$ Laver forcing $\mathbb{L}$ has the incompatibility shrinking property

$\mathfrak{d} = \mathfrak{c} \implies$ Miller forcing $\mathbb{M}$ has the incompatibility shrinking property

Question

Does $\mathbb{L}$ (or $\mathbb{M}$) have the disjoint maximal antichain property in ZFC?

Proposition (from previous slide)

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$\mathbb{T}$ disjoint maximal antichain prop $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$

Assume there is a fusion argument for $\mathbb{T}$ (in this case, t^0 is a σ-ideal).

Then: CH $\implies$ $\mathbb{T}$ has the incompatibility shrinking property
So: CH $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$

For Laver and Miller forcing, weaker hypotheses are sufficient:

Proposition

$\mathfrak{b} = \mathfrak{c}$ $\implies$ Laver forcing $\mathbb{L}$ has the incompatibility shrinking property
$\mathfrak{d} = \mathfrak{c}$ $\implies$ Miller forcing $\mathbb{M}$ has the incompatibility shrinking property

Question

Does $\mathbb{L}$ (or $\mathbb{M}$) have the disjoint maximal antichain property in ZFC?

Proposition (from previous slide)

$\mathbb{T}$ incompatibility shrinking prop $\implies$ $\mathbb{T}$ disjoint maximal antichain prop

$\mathbb{T}$ disjoint maximal antichain prop $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$

Assume there is a fusion argument for $\mathbb{T}$ (in this case, t^0 is a σ-ideal).

Then: CH $\implies$ $\mathbb{T}$ has the incompatibility shrinking property
So: CH $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$

For Laver and Miller forcing, weaker hypotheses are sufficient:

Proposition

$\mathfrak{b} = \mathfrak{c} \implies$ Laver forcing $\mathbb{L}$ has the incompatibility shrinking property
$\mathfrak{d} = \mathfrak{c} \implies$ Miller forcing $\mathbb{M}$ has the incompatibility shrinking property

Question

Does $\mathbb{L}$ (or $\mathbb{M}$) have the disjoint maximal antichain property in ZFC?

Recall: $\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

$$\mathbb{T} \text{ disjoint maximal antichain prop} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Definition

$\mathbb{T}$ has the selective disjoint antichain property if there is an antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that

- $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$,
- for all $S \in \mathbb{T}$ there is $T \leq S$ such that
  - either $T \leq T_\alpha$ for some $\alpha < \mathfrak{c}$,
  - or $|[T] \cap [T_\alpha]| \leq 1$ for all $\alpha < \mathfrak{c}$.

Theorem

$$\mathbb{T} \text{ selective disjoint antichain property} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Recall: $\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

$$\mathbb{T} \text{ disjoint maximal antichain prop} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Definition

$\mathbb{T}$ has the selective disjoint antichain property if there is an antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that

- $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$,
- for all $S \in \mathbb{T}$ there is $T \leq S$ such that
  - either $T \leq T_\alpha$ for some $\alpha < \mathfrak{c}$,
  - or $|[T] \cap [T_\alpha]| \leq 1$ for all $\alpha < \mathfrak{c}$.

Theorem

$$\mathbb{T} \text{ selective disjoint antichain property} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Recall: $\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

$$\mathbb{T} \text{ disjoint maximal antichain prop} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Definition

$\mathbb{T}$ has the selective disjoint antichain property if there is an antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that

- $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$,
- for all $S \in \mathbb{T}$ there is $T \leq S$ such that
  - either $T \leq T_\alpha$ for some $\alpha < \mathfrak{c}$,
  - or $|[T] \cap [T_\alpha]| \leq 1$ for all $\alpha < \mathfrak{c}$.

Theorem

$$\mathbb{T} \text{ selective disjoint antichain property} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Recall: $\mathbb{T}$ has the disjoint maximal antichain property if there is a maximal antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$.

$$\mathbb{T} \text{ disjoint maximal antichain prop} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

Definition

$\mathbb{T}$ has the selective disjoint antichain property if there is an antichain $(T_\alpha : \alpha < \mathfrak{c})$ in $\mathbb{T}$ such that

- $[T_\alpha] \cap [T_\beta] = \emptyset$ for all $\alpha \neq \beta$,
- for all $S \in \mathbb{T}$ there is $T \leq S$ such that
  - either $T \leq T_\alpha$ for some $\alpha < \mathfrak{c}$,
  - or $|[T] \cap [T_\alpha]| \leq 1$ for all $\alpha < \mathfrak{c}$.

Theorem

$$\mathbb{T} \text{ selective disjoint antichain property} \implies cf(\mathrm{cof}(t^0)) > \mathfrak{c}$$

The following property implies that $\mathbb{T}$ adds a minimal real;
in fact, standard proofs of minimality go via this property.

Definition

$\mathbb{T}$ has the constant or one-to-one property if for all $S \in \mathbb{T}$ and all continuous $f : [S] \to 2^\omega$, there is $T \leq S$ such that $f \restriction [T]$ is either constant or one-to-one.

Theorem (in ZFC)

(implicit in Miller) Miller forcing $\mathbb{M}$ has the constant or one-to-one prop
(implicit in Gray) Laver forcing $\mathbb{L}$ has the constant or one-to-one prop

Proposition

$\mathbb{T}$ constant or one-to-one prop $\implies$ $\mathbb{T}$ selective disjoint antichain prop

Recall: $\mathbb{T}$ selective disjoint antichain property $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$
So: $\mathrm{ZFC} \vdash$ $cf(\mathrm{cof}(\ell^0)) > \mathfrak{c}$ and $cf(\mathrm{cof}(m^0)) > \mathfrak{c}$

The following property implies that $\mathbb{T}$ adds a minimal real; in fact, standard proofs of minimality go via this property.

Definition

$\mathbb{T}$ has the constant or one-to-one property if for all $S \in \mathbb{T}$ and all continuous $f : [S] \to 2^\omega$, there is $T \leq S$ such that $f \restriction [T]$ is either constant or one-to-one.

Theorem (in ZFC)

(implicit in Miller) Miller forcing $\mathbb{M}$ has the constant or one-to-one prop
(implicit in Gray) Laver forcing $\mathbb{L}$ has the constant or one-to-one prop

Proposition

$\mathbb{T}$ constant or one-to-one prop $\implies$ $\mathbb{T}$ selective disjoint antichain prop

Recall: $\mathbb{T}$ selective disjoint antichain property $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$
So: $\mathrm{ZFC} \vdash cf(\mathrm{cof}(\ell^0)) > \mathfrak{c}$ and $cf(\mathrm{cof}(m^0)) > \mathfrak{c}$

The following property implies that $\mathbb{T}$ adds a minimal real; in fact, standard proofs of minimality go via this property.

Definition

$\mathbb{T}$ has the constant or one-to-one property if for all $S \in \mathbb{T}$ and all continuous $f : [S] \to 2^\omega$, there is $T \leq S$ such that $f \restriction [T]$ is either constant or one-to-one.

Theorem (in ZFC)

(implicit in Miller) Miller forcing $\mathbb{M}$ has the constant or one-to-one prop
(implicit in Gray) Laver forcing $\mathbb{L}$ has the constant or one-to-one prop

Proposition

$\mathbb{T}$ constant or one-to-one prop $\implies$ $\mathbb{T}$ selective disjoint antichain prop

Recall: $\mathbb{T}$ selective disjoint antichain property $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$
So: $\mathrm{ZFC} \vdash cf(\mathrm{cof}(\ell^0)) > \mathfrak{c}$ and $cf(\mathrm{cof}(m^0)) > \mathfrak{c}$

The following property implies that $\mathbb{T}$ adds a minimal real; in fact, standard proofs of minimality go via this property.

Definition

$\mathbb{T}$ has the constant or one-to-one property if for all $S \in \mathbb{T}$ and all continuous $f : [S] \to 2^\omega$, there is $T \leq S$ such that $f \restriction [T]$ is either constant or one-to-one.

Theorem (in ZFC)

(implicit in Miller) Miller forcing $\mathbb{M}$ has the constant or one-to-one prop
(implicit in Gray) Laver forcing $\mathbb{L}$ has the constant or one-to-one prop

Proposition

$\mathbb{T}$ constant or one-to-one prop $\implies$ $\mathbb{T}$ selective disjoint antichain prop

Recall: $\mathbb{T}$ selective disjoint antichain property $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$
So: $\mathrm{ZFC} \vdash cf(\mathrm{cof}(\ell^0)) > \mathfrak{c}$ and $cf(\mathrm{cof}(m^0)) > \mathfrak{c}$

The following property implies that $\mathbb{T}$ adds a minimal real; in fact, standard proofs of minimality go via this property.

Definition

$\mathbb{T}$ has the constant or one-to-one property if for all $S \in \mathbb{T}$ and all continuous $f : [S] \to 2^\omega$, there is $T \leq S$ such that $f \restriction [T]$ is either constant or one-to-one.

Theorem (in ZFC)

(implicit in Miller) Miller forcing $\mathbb{M}$ has the constant or one-to-one prop
(implicit in Gray) Laver forcing $\mathbb{L}$ has the constant or one-to-one prop

Proposition

$\mathbb{T}$ constant or one-to-one prop $\implies$ $\mathbb{T}$ selective disjoint antichain prop

Recall: $\mathbb{T}$ selective disjoint antichain property $\implies$ $cf(\mathrm{cof}(t^0)) > \mathfrak{c}$
So: $\mathrm{ZFC} \vdash$ $cf(\mathrm{cof}(\ell^0)) > \mathfrak{c}$ and $cf(\mathrm{cof}(m^0)) > \mathfrak{c}$

We do not know of any counterexamples:

Question

Are there combinatorial tree forcings $\mathbb{T}$

1. which consistently fail to have the disjoint maximal antichain prop?
2. which consistently fail to satisfy $\mathrm{cof}(t^0) > \mathfrak{c}$?
3. for which t^0 consistently has a Borel basis?

Even for the following "test case" we do not know anything:
Let fm^0 be the ideal associated to full splitting Miller forcing $\mathbb{FM}$:
$T \in \mathbb{FM}$ if $T \subseteq \omega^{<\omega}$ is a Miller tree such that whenever $s \in T$ is a splitting node, $s^\frown n \in T$ for all $n \in \omega$.

Question

Is $\mathrm{cof}(fm^0) > \mathfrak{c}$ in ZFC? At least no Borel basis in ZFC?

(True under CH.)

We do not know of any counterexamples:

Question

Are there combinatorial tree forcings $\mathbb{T}$

1. which consistently fail to have the disjoint maximal antichain prop?
2. which consistently fail to satisfy $\mathrm{cof}(t^0) > \mathfrak{c}$?
3. for which t^0 consistently has a Borel basis?

Even for the following "test case" we do not know anything:
Let fm^0 be the ideal associated to full splitting Miller forcing $\mathbb{FM}$:
$T \in \mathbb{FM}$ if $T \subseteq \omega^{<\omega}$ is a Miller tree such that whenever $s \in T$ is a splitting node, $s^\frown n \in T$ for all $n \in \omega$.

Question

Is $\mathrm{cof}(fm^0) > \mathfrak{c}$ in ZFC? At least no Borel basis in ZFC?

(True under CH.)

We do not know of any counterexamples:

Question

Are there combinatorial tree forcings $\mathbb{T}$

1. which consistently fail to have the disjoint maximal antichain prop?
2. which consistently fail to satisfy $\mathrm{cof}(t^0) > \mathfrak{c}$?
3. for which t^0 consistently has a Borel basis?

Even for the following "test case" we do not know anything:
Let fm^0 be the ideal associated to full splitting Miller forcing $\mathbb{FM}$:
$T \in \mathbb{FM}$ if $T \subseteq \omega^{<\omega}$ is a Miller tree such that whenever $s \in T$ is a splitting node, $s^\frown n \in T$ for all $n \in \omega$.

Question

Is $\mathrm{cof}(fm^0) > \mathfrak{c}$ in ZFC? At least no Borel basis in ZFC?

(True under CH.)

We do not know of any counterexamples:

Question

Are there combinatorial tree forcings $\mathbb{T}$

1. which consistently fail to have the disjoint maximal antichain prop?
2. which consistently fail to satisfy $\operatorname{cof}(t^0) > \mathfrak{c}$?
3. for which t^0 consistently has a Borel basis?

Even for the following "test case" we do not know anything:

Let fm^0 be the ideal associated to full splitting Miller forcing $\mathbb{FM}$:

$T \in \mathbb{FM}$ if $T \subseteq \omega^{<\omega}$ is a Miller tree such that whenever $s \in T$ is a splitting node, $s^\frown n \in T$ for all $n \in \omega$.

Question

Is $\operatorname{cof}(fm^0) > \mathfrak{c}$ in ZFC? At least no Borel basis in ZFC?

(True under CH.)

We do not know of any counterexamples:

Question

Are there combinatorial tree forcings $\mathbb{T}$

1. which consistently fail to have the disjoint maximal antichain prop?
2. which consistently fail to satisfy $\mathrm{cof}(t^0) > \mathfrak{c}$?
3. for which t^0 consistently has a Borel basis?

Even for the following "test case" we do not know anything:
Let fm^0 be the ideal associated to full splitting Miller forcing $\mathbb{FM}$:
$T \in \mathbb{FM}$ if $T \subseteq \omega^{<\omega}$ is a Miller tree such that whenever $s \in T$ is a splitting node, $s^\frown n \in T$ for all $n \in \omega$.

Question

Is $\mathrm{cof}(fm^0) > \mathfrak{c}$ in ZFC? At least no Borel basis in ZFC?

(True under CH.)

We do not know of any counterexamples:

Question

Are there combinatorial tree forcings $\mathbb{T}$

1. which consistently fail to have the disjoint maximal antichain prop?
2. which consistently fail to satisfy $\mathrm{cof}(t^0) > \mathfrak{c}$?
3. for which t^0 consistently has a Borel basis?

Even for the following "test case" we do not know anything:
Let fm^0 be the ideal associated to full splitting Miller forcing $\mathbb{FM}$:
$T \in \mathbb{FM}$ if $T \subseteq \omega^{<\omega}$ is a Miller tree such that whenever $s \in T$ is a splitting node, $s^\frown n \in T$ for all $n \in \omega$.

Question

Is $\mathrm{cof}(fm^0) > \mathfrak{c}$ in ZFC? At least no Borel basis in ZFC?

(True under CH.)

It is known that $\mathrm{cof}(s^0)$ can consistently assume arbitrary values $\leq 2^{\mathfrak{c}}$ whose cofinality is larger than $\mathfrak{c}$ (Judah-Miller-Shelah) and it is easy to see that the same arguments work for other tree ideals like m^0 and ℓ^0. (In these models CH holds.)

Question

Can we consistently separate the cofinalities of different tree ideals? E.g., are $\mathrm{cof}(s^0) < \mathrm{cof}(m^0)$ or $\mathrm{cof}(m^0) < \mathrm{cof}(s^0)$ consistent?

It is known that $\mathrm{cof}(s^0)$ can consistently assume arbitrary values $\leq 2^{\mathfrak{c}}$ whose cofinality is larger than $\mathfrak{c}$ (Judah-Miller-Shelah) and it is easy to see that the same arguments work for other tree ideals like m^0 and ℓ^0. (In these models CH holds.)

Question

Can we consistently separate the cofinalities of different tree ideals? E.g., are $\mathrm{cof}(s^0) < \mathrm{cof}(m^0)$ or $\mathrm{cof}(m^0) < \mathrm{cof}(s^0)$ consistent?

Thank you for your attention and enjoy the Winter School...

Hejnice 2011

Thank you for your attention and enjoy the Winter School...

Hejnice 2011

Thank you for your attention and enjoy the Winter School. . .

Hejnice 2011