

Cofinalities of Marczewski-type ideals

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In my talk, I would like to present recently finished joint work about the cofinalities of ideals related to various Axiom A tree forcing notions.

It has been known for quite a long time that the cofinality of the "classical" Marczewski ideal s_0 is strictly above the continuum in ZFC. The Marczewski ideal has been introduced way before the beginning of the forcing era, but can now be considered as derived from Sacks forcing: a set X is in the ideal s_0 if and only if for each condition in Sacks forcing there is a stronger condition such that its body (i.e., the set of branches) is disjoint from X . Since Sacks forcing allows fusion, it is actually a σ -ideal.

There are many other well-known tree forcings which are non-ccc but have fusion such as Mathias forcing, Silver forcing, Laver forcing, or Miller forcing. The respective ideals have been systematically studied by Jörg Brendle in his paper "Strolling through paradise" where he shows among other things that all these ideals are mutually different.

In my talk, I will give an overview about our general strategy to prove that the cofinality of any ideal related to a forcing notion mentioned above is indeed larger than the continuum in ZFC. For Sacks, Mathias, and Silver forcing the proof is quite easy; moreover, there is a much simpler argument under CH. So the most interesting case is Laver and Miller forcing without any additional assumption.

If there is time left, I will also discuss open questions: in particular, the case of Full Miller tree forcing (recently considered by Giorgio Laguzzi and Yurii Khomskii) for which our approach does not work, as well as the quest for other (perhaps more exotic) counterexamples.