Let \( B_1(X) \) be the collection of all Baire-one functions on a topological space \( X \).

A subspace \( E \) of a topological space \( X \) is called \( B_1 \)-embedded (\( B_1^* \)-embedded) in \( X \), if any (bounded) function \( f \in B_1(E) \) can be extended to \( g \in B_1(X) \); \( 1 \)-embedded in \( X \), if any functionally \( G_\delta \)-set in \( E \) can be extended to a functionally \( G_\delta \)-set in \( X \); ambiguously \( 1 \)-embedded in \( X \), if any functionally ambiguous set in \( E \) can be extended to a functionally ambiguous set in \( X \); well \( 1 \)-embedded in \( X \), if for any functionally \( G_\delta \)-set \( A \subseteq X \) disjoint with \( E \) there exists a function \( f \in B_1(X) \) such that \( E \subseteq f^{-1}(0) \) and \( A \subseteq f^{-1}(1) \).

We show that a subspace \( E \) of a topological space \( X \) is \( B_1^* \)-embedded in \( X \) if and only if \( E \) is ambiguously \( 1 \)-embedded in \( X \). We prove that \( E \) is \( B_1 \)-embedded in \( X \) if and only if \( E \) is \( 1 \)-embedded and well \( 1 \)-embedded in \( X \). Moreover, any countable hereditarily irresolvable completely regular space is \( B_1^* \)-embedded in \( \beta X \) and is not \( B_1 \)-embedded in \( \beta X \).

Recall that a function \( f : X \to \mathbb{R} \) is fragmented if for every \( \varepsilon > 0 \) and for every closed nonempty set \( F \subseteq X \) there exists a nonempty relatively open set \( U \subseteq F \) such that \( \text{diam} f(U) < \varepsilon \). Notice that every Baire-one real-valued function defined on a hereditarily Baire space is fragmented. We prove that any fragmented function defined on a countable completely regular space \( X \) can be extended to a Baire-one function defined on \( \beta X \).