Maximal trees on $\mathcal{P}(\omega)/fin$

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This is joint work with Michael Hrušák, Gabriela Campero and Favio Miranda

Conventions.

- As usual, instead of working with $\mathcal{P}(\omega)/fin$, we will be working with $[\omega]^\omega$.
- Our trees grow downward.
- Making abuse of notation, we will make reference to trees on $[\omega]^\omega$ as trees on $\mathcal{P}(\omega)/fin$, and viceverse.
- We consider $[\omega]^\omega$ odered by the almost contention $\subseteq^*$: given $A, B \in [\omega]^\omega$, we say that $A \subseteq^* B$ if and only if $A \setminus B$ is finite.

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Trees on $\mathcal{P}(\omega)/fin$

Definition

A tree $\mathcal{T}$ on $\mathcal{P}(\omega)/fin$ is a family of elements of $\mathcal{P}(\omega)/fin$, such that for all $A \in \mathcal{T}$, the set $pred_{\mathcal{T}}(A)$ is well oredered by $\supseteq^*$, the reverse ordering of $\subseteq^*$.

$pred_{\mathcal{T}} = \{B \in \mathcal{T} : A \subseteq^* B\}$ is the set of predecesors of A in the tree $\mathcal{T}$.

Definition

Given two trees T, S on $\mathcal{P}(\omega)/fin$, let us say that $T \sqsubseteq S$ if and only if S is an end extension of T, that is, for every $x \in T$, the sets $pred_T(x) = pred_S(x)$.

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Then the set of all trees on $\mathcal{P}(\omega)/fin$, ordered by $\sqsubseteq$, satisfies the conditions of Zorn's Lemma, so this ordering has maximal elements.

Definition(D. Monk)

Define the cardinal invariant $\mathfrak{tr}$ as the minimum posible size of a maximal tree on $\mathcal{P}(\omega)/fin$, that is,

$$\mathfrak{tr} = \min\{|\mathcal{T}| : \mathcal{T} \subseteq \mathcal{P}(\omega)/fin \text{ is a maximal tree}\}$$

Monk's notation differs from ours. Given a boolean algebra $\mathbb{B}$ he writes $Inc_{mm}^{tree}(\mathbb{B})$ to denote the minimum cardinality of a tree on the boolean algebra $\mathbb{B}$.

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How does a maximal tree on $\mathcal{P}(\omega)/fin$ look like?

Lemma

A tree $\mathcal{T} \subseteq [\omega]^\omega$ is a maximal tree if and only if for every set $A \in [\omega]^\omega$, one of the following holds:

- There is $B \in \mathcal{T}$ such that $B \subseteq^* A$.
- There are $B, C \in \mathcal{T}$ incomparable such that $A \subseteq^* B \cap C$.

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Remark.

If $\mathcal{T}$ is a maximal tree on $\mathcal{P}(\omega)/fin$, then the following family is a reaping family

$$\mathcal{T} \cup \{\omega \setminus A : A \in \mathcal{T}\}$$

So in particular the reaping number is a lower bound for $\mathfrak{tr}$.

Question, D. Monk

Is $\mathfrak{tr} = \mathfrak{c}$?

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Parametrized Diamond Principles

- This are guessing principles which are weakenings of the well known Jensen's diamond principle.
- For each Borel cardinal invariant corresponds a parametrized diamond principle.
- They are compatible with the negation of CH.
- They hold in many of the well known models of set theory.

The $\diamondsuit$-like principle we are using is the corresponding to the cardinal invariant $\mathfrak{r}_\sigma ; \mathfrak{d}$, the sequential composition of $\mathfrak{r}_\sigma$ followd by $\mathfrak{d}$.

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We have found two different shapes for these kind of trees:

Theorem

$\diamondsuit_{L(\mathbb{R})}(\mathfrak{r}_\sigma;\mathfrak{d})$ implies that there is a maximal tree on $\mathcal{P}(\omega)/fin$ of cardinality ω_1, has height ω_1, and all nodes, except the root of the tree(who has ω_1 succesors), have exactly one succesor.

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In the Sacks model $\mathfrak{tr}$ is ω_1, while the continuum is ω_2.

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In the construction of the trees in the two theorems, we make use of the dominating number, and it is not clear how to skip this, so one may ask wheather the dominating number $\mathfrak{d}$ is a lower bound of $\mathfrak{tr}$.

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Is $\mathfrak{d}$ a lower bound for $\mathfrak{tr}$?

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The tree of height ω mentioned above is actually an ideal-tree.

Proposition

Let $\mathcal{T}$ be a maximal tree on $\mathcal{P}(\omega)/fin$. Then

- If $\mathcal{T}$ is an ideal-tree, then it has size at least $\mathfrak{d}$.
- If $\mathcal{T}$ has a branch of countable cofinality, then $\mathfrak{d} \leq |\mathcal{T}|$.
- If $\mathcal{T}$ has an infinite AD family then $\mathfrak{d} \leq |\mathcal{T}|$.
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Theorem

It is consistent that $\mathfrak{tr} < \text{non}(\mathcal{M})$. In particular it is consistent $\mathfrak{tr} < \mathfrak{i}$.

Guideline of proof:

- Make a ω_2-length CSI of any of your favourite fat tree forcing.
- This forcing is ω^ω-bounding and adds eventually different reals.
- It was proved by J. Zapletal that this forcing preserves Ramsey ultrafilters.

So in the final extension the σ-reaping number and the dominating number are both ω_1, meanwhile $\text{non}(\mathcal{M})$ is big. Since this forcing is a definable forcing notion, it follows that $\diamondsuit_{L(\mathbb{R})}(\mathfrak{r}_\sigma, \mathfrak{d})$ holds.

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