Luzin π-bases and the foliage hybrid operation

Foliage trees

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Recall that the Baire space $\mathcal{N}$ is ${}^{\omega}\omega$ with the product topology.

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There is a Baire foliage tree on the Sorgenfrey line $\mathcal{R}_{\mathcal{S}}$.

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- **B** is a Luzin π-base for $\mathcal{R}_{\mathcal{S}}$.

Luzin π-base

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Let $\mathsf{LPB} :=$ the class of spaces that have a Luzin π-base.

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- Up to homeomorphisms,
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- if $X \in \mathrm{LPB}$,
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Luzin π-base

Examples

The following spaces lie in LPB:

- The irrational Sorgenfrey line $\mathcal{I}_\mathcal{S}$;
- if $X_\alpha \in \{\mathcal{N}, \mathcal{R}_\mathcal{S}, \mathcal{I}_\mathcal{S}\}$ and $0 < |A| \le \aleph_0$,
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- if $X \in$ LPB and $F \subseteq X$ is a σ-compact,
  then $X \setminus F \in$ LPB (the proof uses the *foliage hybrid operation*).

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hybrid$(\mathcal{T}, \gamma)$ is

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Proposition

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Proposition

hybrid$(\mathcal{T}, \gamma)$ is a tree.

Foliage hybrid operation

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✎ $\mathrm{cut}(\mathbf{F}, \mathbf{G}) := \mathbf{F}_{0_\mathbf{G}} \setminus \mathbf{G}_{0_\mathbf{G}}$;

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- If $\mathbf{F}$ has strict branches and splittable, and if each $\mathbf{G} \in \gamma$ has bounded chains, then fol.hybrid$(\mathbf{F}, \gamma)$ has strict branches.

How to build Luzin π-base for a subspace of X

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Let $\mathbf{L}$ be a Luzin π-base for X

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If $\mathbf{L}$ grows into X and each $\mathbf{G} \in \gamma$ preserves shoots of $\mathbf{L}$,

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Proposition

If $\mathbf{L}$ grows into X and each $\mathbf{G} \in \gamma$ preserves shoots of $\mathbf{L}$, then fol.hybrid$(\mathbf{L}, \gamma)$ grows into $X \setminus \text{loss}(\mathbf{L}, \gamma)$.

M. Patrakeev, *The complement of a σ-compact subset of a space with a Luzin π-base also has a Luzin π-base*, preprint.
http://arxiv.org/abs/1512.02458

Thank you!