Abstract of my talk

Wolfgang Wohofsky

My research area is (iterated) forcing and its applications to set theory of the reals; in particular, I study questions about small (or: "special") sets of real numbers (and variants of the Borel Conjecture).

A prominent example is the class of *strong measure zero* (smz) sets (a set X is smz if for any sequence of ε_n's, X can be covered by intervals I_n of length ε_n). The *Galvin-Mycielski-Solovay theorem* states that X is smz if and only if it is *meager-shiftable* (i.e., if it can be translated away from each meager set).

The *Borel Conjecture* (BC) is the statement that there are no uncountable smz sets; the *dual Borel Conjecture* (dBC) is the analogous statement about *strongly meager* sets (the sets which are *null-shiftable*). Both BC and dBC fail under CH. In 1976, Laver showed that BC is consistent; Carlson showed that dBC is consistent (actually it holds in the Cohen model).

In my thesis, I started investigating another variant of the Borel Conjecture (which I called the *Marczewski Borel Conjecture*, MBC). It is the assertion that there is no uncountable s_0*-shiftable* set (the *Marczewski ideal* s_0 is related to Sacks forcing: a set Z is in s_0 if each perfect set contains a perfect subset disjoint from Z). I introduced the notion of *Sacks dense ideal* (being a translation-invariant σ-ideal $\mathcal{I} \subseteq \mathcal{P}(2^\omega)$ which is "dense in Sacks forcing", i.e., each perfect set contains a perfect subset which belongs to $\mathcal{I}$) to investigate whether MBC is consistent.

In my talk I will present quite recent work joint with Jörg Brendle which in particular shows that MBC is consistent (by actually showing that it follows from CH). More generally, we prove the following theorem in ZFC:

> There is no s_0-shiftable set of size $2^{\aleph_0}$.

To prove this we use the following notion: let us say that a set $X \subseteq 2^\omega$ is $<\kappa$*-Hejnice* if for each perfect set P there exists a perfect subset $Q \subseteq P$ such that for any translate $Q + t$ of Q we have $|(Q + t) \cap X| < \kappa$. It is quite easy to show (for regular continuum) that every set X of size continuum that is $<2^{\aleph_0}$-Hejnice is not s_0-shiftable. The above mentioned theorem is then obtained by showing in ZFC that any continuum sized set contains a continuum sized set in s_0 (in other words, there is no Luzin type set with respect to s_0) which in turn contains a continuum sized set which is $<2^{\aleph_0}$-Hejnice. In fact, in case the continuum is singular, we have to show a bit more: the resulting set is not only $<2^{\aleph_0}$-Hejnice, but even $<\kappa$-Hejnice for some $\kappa < 2^{\aleph_0}$.