Systems of Filters

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(joint work with Giorgio Audrito)

Generic large cardinal axioms postulate the existence of elementary embeddings $j : V \to M$ with M a transitive class definable in some forcing extension of V (see e.g.[3]). The main feature of the generic large cardinals is that, contrary to the classical case, one can consistently have generic large cardinal properties at cardinals as small as ω_1.

Due to the *class* nature of the elementary embeddings involved in the definitions of large cardinals (both classical and generic), a key issue concerns the possibility to define (or derive) such embeddings from set-sized objects. The first natural candidate are ideals, although it turns out that they are not able to represent various relevant large cardinal properties. For this reason many extensions of the concept have been proposed, the most important of which are extenders (see e.g.[4, 5]) and normal towers (see e.g. [2, 6]).

We introduce the notion of *$\mathcal{C}$-system of filters*. This concept is inspired by the definitions of extenders and towers of normal ideals, generalizes both of them, and provides a common framework in which the standard properties of extenders and towers used to define classical or generic large cardinals can be expressed in an elegant and concise way. Using the new framework given by $\mathcal{C}$-system of filters we easily generalize to the setting of generic large cardinals well-known results about extenders and towers, providing shorter and modular proofs of several facts regarding classical and generic large cardinals. Furthermore, we are able to examine closely the relationship between extenders and towers, and investigate when they are equivalent or not, both in the standard case and in the generic one.

References

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