Abstract

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In recent years set theory fruitfully approached functional analysis, and in particular the theory of C^*–algebras (see [1] and [2]).

In my thesis I try to outline some analogies between these two theories, working with C^*-algebras and boolean valued extensions of the complex field. In fact, some specific C^*–algebras can be studied in the context of boolean valued models appealing to Gelfand Transform: given a commutative unital C^*–algebra A with extremely disconnected spectrum, there is an isomorphism (which can be defined using the Gelfand Transform) of the C^*–algebras A and $C(St(\mathsf{B}))$ (which can be thought as a boolean valued extension of the complex field), where B is the boolean algebra given by clopen sets in the weak* topology on the spectrum of A. By means of this isomorphism A can be therefore embedded in the set of B-names for complex numbers in the boolean model V^{B}.

This embedding might be an interesting tool to translate ideas and results arising in set theory to ideas and results arising in the study of commutative C^*–algebras and conversely.

An interesting development of this might follow using the Shoenfield absoluteness theorem in order to carry properties from the theory of C^*-algebras, seen as boolean valued models, to the first order theory of complex numbers and vice versa.

References

1. Nik Weaver. Set Theory and C^*–algebras. In *The Bulletin of Symbolic Logic*, Volume 13, pages 1-20. 2007.

2. Ilijas Farah and Eric Wofsey. Set Theory and Operator Algebras. In James Cummings and Ernest Schimmerling, editors, *Appalachian Set Theory 2006-2012*, pages 63-120. Cambridge University Press. 2012.