Local function vs. local closure function

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If $\langle X, \tau \rangle$ is a topological space and $\mathcal{I}$ an ideal on X, then a triple $\langle X, \tau, \mathcal{I} \rangle$ is called an ideal topological space. If $\langle X, \tau, \mathcal{I} \rangle$ is an ideal topological space, then the mapping $A \mapsto A^*$ defined by

$$A^* = \{x \in X : A \cap U \notin \mathcal{I} \text{ for each } U \in \tau(x)\}$$

is called the local function (see [2]).

A set U is θ-open iff for each $x \in U$ there exists $V \in \tau(x)$ such that $\mathrm{Cl}(V) \subseteq U$ (see [4]). A set A is θ-closed iff $X \setminus A$ is θ-open. Also, a set A is θ-closed iff it is equal to its θ-closure $\mathrm{Cl}_\theta(A)$ defined by

$$\mathrm{Cl}_\theta(A) = \{x \in X : \mathrm{Cl}(U) \cap A \neq \emptyset \text{ for each } U \in \tau(x)\}.$$

Generalizing the notions of local function and θ-closure, Al-Omari and Noiri [1] in an ideal topological space $\langle X, \tau, \mathcal{I} \rangle$ defined the **local closure function** which maps a set A into $\Gamma_{(\tau,\mathcal{I})}(A)$, where

$$\Gamma_{(\tau,\mathcal{I})}(A) = \{x \in X : \mathrm{Cl}(U) \cap A \notin \mathcal{I} \text{ for each } U \in \tau(x)\}.$$

They proved some basic properties for the local closure function, and also introduced two new topologies obtained by the original one using the local closure function.

In this talk, based on [3], the aim is to present properties of the local closure function and also similarities and differences between it and the local function, varying ideal in the topological space by using several well-known ideals, like ideals of finite sets, countable sets, closed and discrete sets, scattered sets, nowhere dense sets and others. Also, a special attention is paid to the class of ideal topological spaces with the ideal of finite sets Fin.

References

[1] A. Al-Omari, T. Noiri, Local closure functions in ideal topological spaces. Novi Sad J. Math. 43 (2013), no. 2, 139-149.

[2] D. Janković, T.R. Hamlett, New topologies from old via ideals. Amer. Math. Monthly 97 (1990), no. 4, 295-310.

[3] A. Pavlović, Local function versus local closure function in ideal topological spaces, submited.

[4] N.V. Veličko, H-closed topological spaces. Mat. Sb. (N.S.) 70 (112) (1966) 98-112 (in Russian); in: American Mathematical Society Translations, vol. 78, American Mathematical Society, Providence, RI, 1969, 103-118.