Products of Menger spaces

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A topological space X is *Menger* if for every sequence of open covers $\mathcal{O}_1, \mathcal{O}_2, \dots$ there are finite sets $\mathcal{F}_1 \subseteq \mathcal{O}_1, \mathcal{F}_2 \subseteq \mathcal{O}_2, \dots$ such that the family $\{\bigcup \mathcal{F}_1, \bigcup \mathcal{F}_2, \dots\}$ is a cover of X. If we can request that for every element $x \in X$ the set $\{n : x \in \bigcup \mathcal{F}_n\}$ is co-finite, then the space X is *Hurewicz*. The above properties generalize σ-compactness and Hurewicz's property is strictly stronger than Menger's (Chaber-Pol and Tsaban-Zdomskyy).

One of the major open problems in the field of selection principles is whether there are, in ZFC, two Menger sets of real numbers whose product is not Menger. We provide examples under various set theoretic hypotheses, some being weak portions of the Continuum Hypothesis, and some violating it. The proof method is new.

We also consider filter versions of the above-mentioned properties, and prove that they are strictly inbetween Hurewicz and Menger.

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