On maximal connected topologies

Adam Bartoš

Abstract

If $\mathcal{P}$ is a property of topological spaces, we say that a topological space $\langle X, \tau \rangle$ (or the corresponding topology τ) is *maximal* $\mathcal{P}$ if it has the property $\mathcal{P}$, but no strictly finer topology $\tau^* > \tau$ has the property $\mathcal{P}$. We are interested in the case where $\mathcal{P}$ means connectedness, i.e. in *maximal connected topologies*. There are examples of Hausdorff maximal connected spaces as well as examples of Hausdorff connected spaces having no finer topology that is maximal connected. The problem of existence of a regular maximal connected topology remains open.

We will summarize the history of the problem of existence of maximal connected spaces with prescribed separation axioms, present known facts, and illustrate them on several examples. We will also present our result that maximal connectedness is preserved by so-called tree sums in the case that the gluing set is closed discrete.