The principle (*) of Sierpinski and non-meager sets
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The principle (*) of Sierpinski is the assertion that there is a family of functions \( \{ \varphi_n : \omega_1 \rightarrow \omega_1 \mid n \in \omega \} \) such that for every \( I \in [\omega_1]^\omega \) there is \( n \in \omega \) such that \( \varphi_n [I] = \omega_1 \). We prove that this principle holds if there is a non-meager set of size \( \omega_1 \), answering question of Arnie Miller. Combining our result with a theorem of Miller it then follows that (*) is equivalent to \( \non (M) = \omega_1 \). Miller also proved that the principle of Sierpinski is equivalent to the existence of a weak version of a Luzin set, we will construct a model where all of this sets are meager yet \( \non (M) = \omega_1 \).