Rosenthal compacta that are premetric of finite degree

Alejandro Poveda

joint work with Antonio Avilés and Stevo Todorcevic

A compact space K is said to be Rosenthal if it is homeomorphic to a pointwise compact space in $\boldsymbol{\mathcal{B}}_1(X)$ where X is a polish space. Rosenthal compacta have become an important research topic on Functional Analysis after the well-known Odell-Rosenthal's characterization of separable Banach spaces without copies of ℓ^1.

Despite its analytical origins, Rosenthal compacta have been extensively studied from other areas like general topology and its study involves techniques from Logic, Ramsey Theory and Descriptive Set Theory. In this regard we can highlight a celebrated result on the structure of the class of separable Rosenthal compacta due to S. Todorcevic [Tod99]. Namely, Todorcevic proved that a separable Rosenthal compactum either contains a discrete subspace of size continuum or it is a *pre-metric compactum of degree at most two*. Moreover, the author proves that such kind of compacta contain either copies of the Split Interval $S(I)$ or copies of the Alexadroff Duplicate of the Cantor space $D(2^{\mathbb{N}})$.

In this work we generalize the idea of pre-metric compactum of degree at most two considering the notion of Rosenthal compactum of finite degree and investigate the structure of this new class of Rosenthal compacta. Specifically, we prove that every separable Rosenthal compactum of degree n but not of degree $n-1$ contain either copies of the $n-$Split Interval $S_n(I)$ or copies of the Alexadroff $n-plicate$ $D_n(2^{\mathbb{N}})$.

References

[Tod99] Stevo Todorčević. Compact subsets of the first baire class. *Journal of the American Mathematical Society*, 12(4):1179–1212, 1999.