ROSENTHAL FAMILIES AND THE GROTHENDIECK PROPERTY

DAMIAN SOBOTA

The Rosenthal lemma, being one of most fundamental results in measure theory, reads as follows:

> *Let* $(a_n : n \in \omega)$ *be an antichain in* $\wp(\omega)$*. Assume* (μ_k) *is a sequence of positive finitely additive measures on* $\wp(\omega)$ *satisfying the inequality* $\mu_k\left(\bigcup_{n\in\omega} a_n\right) < 1$ *for every* $k \in \omega$*. Fix* $\varepsilon > 0$*. Then, there exists an infinite set* $A \subseteq \omega$ *such that for every* $k \in A$ *the following inequality is satisfied:*
>
> $$\mu_k\Big(\bigcup_{\substack{n\in A\\ n\neq k}} a_n\Big) < \varepsilon.$$

Despite its simplicity (and oddity), the lemma is a powerful tool when applied to study of functional and operator properties of Banach spaces.

During the first part of my talk, I shall show how to rephrase the lemma in terms of selective ultrafilters (assuming they exist). This immediately gives a new a version of a classical Rosenthal theorem on embedding ℓ_∞ into an arbitrary Banach space. In the second part, I will show how the results from the first part may be applied to consistently construct an infinite Boolean algebra with the Grothendieck property and of cardinality strictly less than the continuum $\mathfrak{c}$.

(All the necessary definitions will be provided during the talk. No technical proofs will be shown.)

Institute of Mathematics, Polish Academy of Sciences
E-mail address: damian.sobota@impan.pl