Ultrafilters on Semifilters

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Dynamical systems: a very short introduction

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- For example, if (and only if) X is connected then (X, id_X) is chain transitive. For another (more important) example:

Theorem

ω^* is chain transitive.

Chain transitivity is important

In fact, the fact that ω^* is chain transitive seems somehow to capture the main features of its dynamical structure:

Theorem

If X is a metrizable dynamical system, then X is a quotient of ω^* if and only if X is chain transitive.

Theorem

Assuming $\mathrm{MA}_{\sigma\text{-centered}}$, this extends to all X with $w(X) < \mathfrak{c}$.

Theorem

It is consistent with and independent of ZFC that the shift map and its inverse are the only chain transitive autohomeomorphisms ω^*.

filters and friends

A *filter* $\mathcal{F}$ on a partial order $\langle \mathbb{P}, \leq \rangle$ is a subset of $\mathbb{P}$ satisfying:

1. **Nontriviality:** $\emptyset \neq \mathcal{F}$.
2. **Upwards heredity:** if $a \in \mathcal{F}$ and $a \leq b$, then $b \in \mathcal{F}$.
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$\mathcal{F}$ is an *ultrafilter* if it satisfies $(1) - (3)$ and

4. **Maximality:** no proper superset of $\mathcal{F}$ is a filter.

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- The set of ultrafilters on $\mathcal{P}(\omega)/\mathrm{Fin}$ has a naturally topology making it the Čech-Stone compactification of ω, denoted ω^*.
- Every filter $\mathcal{F}$ on $\mathcal{P}(\omega)/\mathrm{Fin}$ corresponds to a closed subset $\hat{\mathcal{F}}$ of ω^*, and $\mathcal{F} \subseteq \mathcal{G}$ iff $\hat{\mathcal{G}} \subseteq \hat{\mathcal{F}}$. This correspondence is a special case of what is called *Stone duality*.

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- *$\mathcal{F}$ is an ultrafilter on Θ.*
- *$\hat{\mathcal{F}}$ is a minimal dynamical subsystem of (ω^*, σ).*
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Thus understanding the ultrafilters on Θ helps us to understand the canonical dynamical and algebraic structures on ω^*.

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Proof.

Maybe you should ask Justin . . . □

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Recall that any subset of ω can be identified with an element of 2^ω (via characteristic functions). Thus a semifilter on $\mathcal{P}(\omega)/\mathrm{Fin}$ can be identified with a subset of 2^ω.

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If $\mathfrak{F}$ is a semifilter and is G_δ in 2^ω, then $\mathfrak{p}_\mathfrak{F} = \mathfrak{t}_\mathfrak{F}$.

Remark: The requirement that $\mathfrak{F}$ be G_δ cannot be relaxed: there is an F_σ semifilter $\mathfrak{F}$ such that $\mathfrak{p}_\mathfrak{F} = \mathfrak{t}_\mathfrak{F} = \aleph_0$.

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Proof (not really)

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Since $\mathfrak{p} = \mathfrak{t}$, it is enough to show that $\mathfrak{p} \leq \mathfrak{p}_{\mathfrak{F}} \leq \mathfrak{t}_{\mathfrak{F}} \leq \mathfrak{t}$. We'll sketch the argument for $\mathfrak{p} \leq \mathfrak{p}_{\mathfrak{F}}$:

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If $\mathfrak{p} = \mathfrak{c}$, then . . .

Theorem

Let $\mathfrak{F}$ be a G_δ semifilter. If $\mathfrak{p} = \mathfrak{c}$, then there is an ultrafilter on $\mathfrak{F}$ that is also a P-filter.

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Proof.

Let $\langle S_\alpha : \alpha < \mathfrak{c} \rangle$ be an enumeration of $\mathfrak{F}$. We construct a (reverse well ordered) chain $\{X_\alpha : \alpha < \mathfrak{c}\}$ in $\mathfrak{F}$ as follows. Set $X_0 = \omega$. If X_α has already been defined, let $X_{\alpha+1} = X_\alpha \cap S_\alpha$ if $X_\alpha \cap S_\alpha \in \mathfrak{F}$, and otherwise let $X_{\alpha+1} = X_\alpha$. For limit α, let X_α be any lower bound in $\mathfrak{F}$ of the chain $\{X_\beta : \beta < \alpha\}$; such a bound exists because $\alpha < \mathfrak{t}_{\mathfrak{F}}$. A chain constructed in this way will be the basis for an ultrafilter on $\mathfrak{F}$, and is clearly a P-filter. □

Example I: cool P-points

Corollary

Suppose $\mathfrak{p} = \mathfrak{c}$. If $\mathfrak{F}$ is a G_δ semifilter that also has the Ramsey property, then there is a P-point $p \in \omega^*$ with $p \subseteq \mathfrak{F}$.

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- There is a P-point p such that every $A \in p$ contains arbitrarily long arithmetic sequences. (Notice that such an ultrafilter is a "down-to-earth" example of a P-point that fails to be selective.)

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- Fix a copy of the Rado graph with ω as the set of vertices. There is a P-point p such that for every $A \in p$, some subset of A is isomorphic to the Rado graph.

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- *the minimal right ideals of ω^* are not homeomorphically embedded.*
- *$(\omega^*, +)$ has prime ideals that are also minimal.*
- *there is an idempotent ultrafilter that is both minimal and right maximal.*
- *assuming CH, there is a chain transitive map on ω^* that is isomorphic to neither the shift map nor its inverse.*

A few questions about semifilters

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Is there a model in which no G_δ semifilter has a P-ultrafilter on it? For which $\mathfrak{F}$ is it possible to keep P-points while eliminating P-ultrafilters on $\mathfrak{F}$? The other way around?

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A positive answer is obviously consistent (just put $\mathfrak{t} = \mathfrak{c}$). Any semifilter that would give a negative answer must be meager in 2^ω. However, if we replace "Borel" with "meager" then a consistent negative answer is already known (in a length-ω_3 finite-support iteration of Hechler forcing over a model of CH).

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If X is a chain transitive dynamical system of weight $\leq \aleph_1$, is it necessarily true that X is a quotient of ω^*?