Ordered sets of Baire class 1 functions

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Let X be a Polish space. The pointwise limits of continuous functions defined on X are called Baire class 1 functions (denoted by $\mathcal{B}_1(X)$). A natural partial ordering on $\mathcal{B}_1(X)$ is the pointwise ordering, that is, we say that $f < g$ if for every $x \in X$ we have $f(x) \leq g(x)$ and there exists an x so that $f(x) < g(x)$. The description of the linearly ordered subsets of $(\mathcal{B}_1(X), <)$ reveals lots of information about the poset $(\mathcal{B}_1(X), <)$. We say that a linearly ordered set $(L, <_L)$ is *embeddable* into a poset $(P, <)$ if P contains an order isomorphic copy of L. It was shown by Kuratowski that ω_1 is not representable in $\mathcal{B}_1(X)$.

In the 70s Laczkovich posed the following problem:

Problem. Characterise the linearly ordered subsets of the poset $(\mathcal{B}_1(X), <)$.

Partial results were proved by Komjáth, Steprāns, Kunen and Elekes concerning this problem. In a joint work with Márton Elekes we solved Laczkovich's problem proving that there exists a concrete, combinatorially describable universal linearly ordered set $(U, <_U)$, that is, a linearly ordered set so that a linearly ordered set is embeddable into $(\mathcal{B}_1(X), <)$ iff it is embeddabble into $(U, <_U)$. Using this result we answered all of the known open questions concerning the linearly ordered subsets of the poset $(\mathcal{B}_1(X), <)$.