Ordered sets of Baire class 1 functions

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Let $X$ be a Polish space. The pointwise limits of continuous functions defined on $X$ are called Baire class 1 functions (denoted by $B_1(X)$). A natural partial ordering on $B_1(X)$ is the pointwise ordering, that is, we say that $f < g$ if for every $x \in X$ we have $f(x) \leq g(x)$ and there exists an $x$ so that $f(x) < g(x)$. The description of the linearly ordered subsets of $(B_1(X), <)$ reveals lots of information about the poset $(B_1(X), <)$. We say that a linearly ordered set $(L, <_L)$ is embeddable into a poset $(P, <)$ if $P$ contains an order isomorphic copy of $L$. It was shown by Kuratowski that $\omega_1$ is not representable in $B_1(X)$.

In the 70s Laczkovich posed the following problem:

**Problem.** Characterise the linearly ordered subsets of the poset $(B_1(X), <)$.

Partial results were proved by Komjáth, Steprāns, Kunen and Elekes concerning this problem. In a joint work with Márton Elekes we solved Laczkovich’s problem proving that there exists a concrete, combinatorially describable universal linearly ordered set $(U, <_U)$, that is, a linearly ordered set so that a linearly ordered set is embeddable into $(B_1(X), <)$ iff it is embeddable into $(U, <_U)$. Using this result we answered all of the known open questions concerning the linearly ordered subsets of the poset $(B_1(X), <)$. 