Game theoretic approach to skeletally Dugundji and Dugundji spaces

by

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Abstract: Let \( g \) and \( \phi \) be two maps defined on a space \( X \). If there exists a map \( h : \phi(X) \to g(X) \) such that \( g = h \circ \phi \), then we write \( \phi \prec g \).

A family \( \Psi \) of maps with a common domain \( X \) is a multiplicative lattice of skeletal (open) maps whenever:

(L0) \( \Psi \) consists of skeletal (open), maps, only;
(L1) For any map \( f : X \to f(X) \) there exists \( \phi \in \Psi \) with \( \phi \prec f \) and \( w(\phi(X)) \leq w(f(X)) \);
(L2) If \( \{\phi_\alpha : \alpha \in J\} \subset \Psi \), then the diagonal map \( \Delta\{\phi_\alpha : \alpha \in J\} \) is homeomorphic to some element of \( \Psi \).

A Tychonoff space \( X \) is called skeletally Dugundji if it has a multiplicative lattice of skeletal maps. A Dugundji space one can define as a Compact Hausdorff space which have a multiplicative latices of open maps. Characterizations of skeletally Dugundji spaces and Dugundji spaces are given in terms of club collections, consisting of countable families of co-zero sets. For example, a Tychonoff space \( X \) is skeletally Dugundji if and only if there exists an additive \( c \)-club on \( X \). Dugundji spaces are characterized by the existence of additive \( d \)-clubs.