Game theoretic approach to skeletally Dugundji and Dugundji spaces

by

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Abstract: Let g and ϕ be two maps defined on a space X. If there exists a map $h : \phi(X) \to g(X)$ such that $g = h \circ \phi$, then we write $\phi \prec g$. A family Ψ of maps with a common domain X is a *multiplicative lattice of skeletal (open) maps* whenever:

- (L0) Ψ consists of skeletal (open), maps, only;
- (L1) For any map $f \colon X \to f(X)$ there exists $\phi \in \Psi$ with $\phi \prec f$ and $w(\phi(X)) \leq w(f(X))$;
- (L2) If $\{\phi_\alpha : \alpha \in \mathbb{J}\} \subset \Psi$, then the diagonal map $\triangle\{\phi_\alpha : \alpha \in \mathbb{J}\}$ is homeomorphic to some element of Ψ.

A Tychonoff space X is called *skeletally Dugundji* if it has a multiplicative lattice of skeletal maps. A Dugundji space one can define as a Compact Hausdorff space which have a multiplicative latices of open maps. Characterizations of skeletally Dugundji spaces and Dugundji spaces are given in terms of club collections, consisting of countable families of co-zero sets. For example, a Tychonoff space X is skeletally Dugundji if and only if there exists an additive c-club on X. Dugundji spaces are characterized by the existence of additive d-clubs.