Avoidable polynomials and $\mathbb{R} \subseteq L$

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In [3, 4] Törnquist and Weiss proved many natural Σ^1_2 definable counterparts of classical equivalences to the Continuum Hypothesis (CH). These become equivalent to "all reals are constructible". Following this scheme, we proved definable counterparts for some algebraic equivalent form of CH.

More specifically we obtained a Σ^1_2 version of a result about avoidable polynomials proven by Schmerl [2]. As a corollary, we have $\mathbb{R} \subseteq L$ if and only if there exists a Σ^1_2 coloring of the plane in countably many colors with no monochromatic right-angled triangle, which is the Σ^1_2 analogous of a famous result by Erdős and Komjáth [1].

References

[1] Paul Erdős and Péter Komjáth. Countable Decompositions of R2 and R3. *Discrete Comput. Geom.*, 5(4):325–331, May 1990.

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[3] Asger Törnquist and William Weiss. Definable Davies' Theorem. *Fundamenta Mathematicae*, 205(1):77–89, 2009.

[4] Asger Törnquist and William Weiss. The Σ^1_2 counterparts to statements that are equivalent to the continuum hypothesis. 2012.