Zero-dimensional spaces as topological and Banach fractals

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A topological space X is called a *topological fractal* if $X = \bigcup_{f \in \mathcal{F}} f(X)$ for a finite system $\mathcal{F}$ of continuous self-maps of X which is *topologically contracting* in the sense that for every open cover $\mathcal{U}$ of X there is a number $n \in \mathbb{N}$ such that for any functions $f_1, \dots, f_n \in \mathcal{F}$ the set $f_1 \circ \cdots \circ f_n(X)$ is contained in some set $U \in \mathcal{U}$. If, in addition, all functions $f \in \mathcal{F}$ has Lipschitz constant < 1 with respect to some metric generating the topology of X, then the space X is called a *Banach fractal*. It is known that each topological fractal is compact and metrizable. We prove that a zero-dimensional compact metrizable space X is a topological fractal if and only if X is a Banach fractal if and only if X is either uncountable or X is countale and its scattered height $\hbar(X)$ is a successor ordinal.