PATTERNS OF STATIONARY REFLECTION

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If κ is a regular cardinal, $\alpha < \kappa$ has uncountable cofinality, and $S \subseteq \kappa$ is stationary, we say S *reflects at* α if $S \cap \alpha$ is stationary at α. S *reflects* if there is $\alpha < \kappa$ such that S reflects at α. Questions regarding the extent of stationary reflection have been extensively studied and are intimately related to a number of topics concerning large cardinals, combinatorial set theory, and cardinal arithmetic. Eisworth, motivated in part by his work on square-bracket partition relations, asked whether it must be the case that if λ is a singular cardinal and every stationary subset of λ^+ reflects, then every stationary subset of λ^+ reflects at ordinals of arbitrarily high cofinality below λ. We will answer this in the negative and go on to consider variants of Eisworth's question. Along the way, we will explore some connections between stationary reflection and the combinatorial notion of approachability.