A Survey of the Density Topology

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Suppose we have a function $f : \mathbb{R} \to \mathbb{R}$ with the following property:

$\exists A \subseteq \mathbb{R}$ such that the Lebesgue measure $m(A) = 0$ and $f \restriction_A$ is continuous.

The collection of all functions satisfying this property is strictly larger than the collection of all continuous functions. We explore some natural extensions of continuity to functions which satisfy this property.

Using Lebesgue measure, we define the density of a point. With Lebesgue's Density Theorem, we know that the density function and characteristic function agree almost everywhere. We then proceed to the notion of *approximate continuity*[1]. With the definition of density, we can describe the density topology and note some of the topological properties. Finally, we have a topology such that functions satisfying the above property are continuous. We pay particular attention to Alessandro Andretta and Riccardo Camerlo's[2] work and Wilczyński's[3] article.

[1] C. Goffman and D. Waterman, *Approximately Continuous Transformations.* Amer. Math Soc. 1961.

[2] Alessandro Andretta and Riccardo Camerlo. *Descriptive Set Theory of the Lebesgue Density Theorem.* www.arxiv.org/_abs/1105.3355v1. May 17, 2011.

[3] W. Wilczyński. "Density Topologies." *Handbook of Set Theory, Vol. 1.,* Elsevier Science, Netherlands, 2002. p. 675-702