# On a new $\mathbf{F}_\sigma$ ideal

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An ideal $\mathcal{I}$ on $\omega$ is *Mon* if every sequence of reals contains a monotone subsequence indexed by an $\mathcal{I}$-positive set.

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# The ideal $\mathcal{K}$

Define a coloring $\chi\colon [\omega \times \omega]^2 \to \{\text{blue}, \text{red}\}$ by:

$$\chi\left((i,j),(k,l)\right) = \begin{cases} \text{blue} & \text{if } k > i + j \\ \text{red} & \text{if } k \leq i + j \end{cases}$$

for $(i,j),(k,l) \in \omega \times \omega$ such that $(i,j) \leq_{\text{lex}} (k,l)$.

**Definition (K.)**

*$\mathcal{K}$ is the ideal generated by $\chi$-homogeneous subsets of $\omega \times \omega$, i.e., sets $H \subset \omega \times \omega$ such that $\chi \restriction [H]^2$ is constant.*

It is immediate that $\mathcal{K}$ is not 2-Ramsey. Moreover one can prove that $\mathcal{K}$ is $\mathbf{F}_\sigma$.

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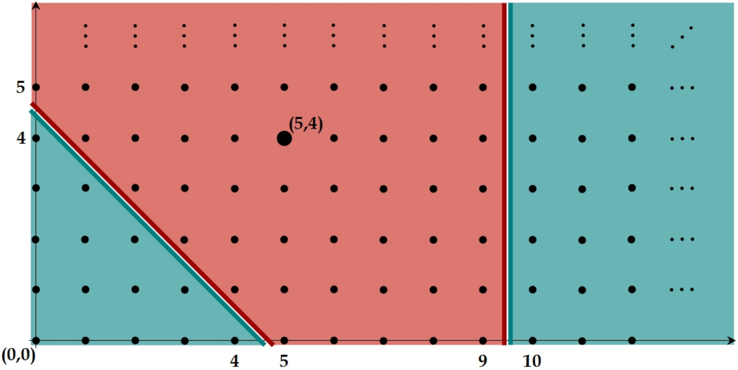
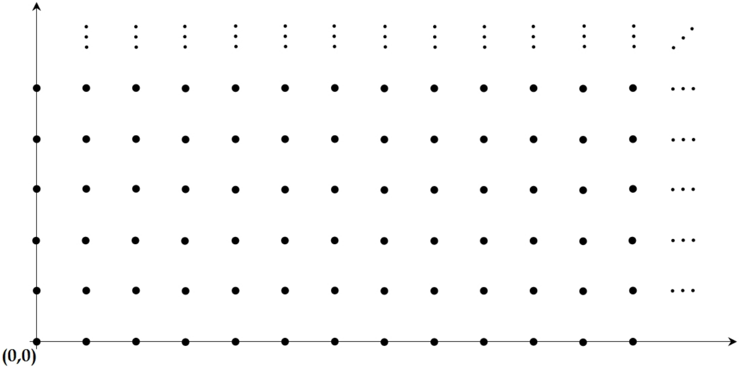
# The ideal $\mathcal{K}$ - some pictures



$\mathcal{K}$ is generated by two kinds of sets.

- all vertical lines, i.e. all sets $\{i\} \times \omega$ for $i \in \omega$;
- subsets of $\omega \times \omega$ of the following form:



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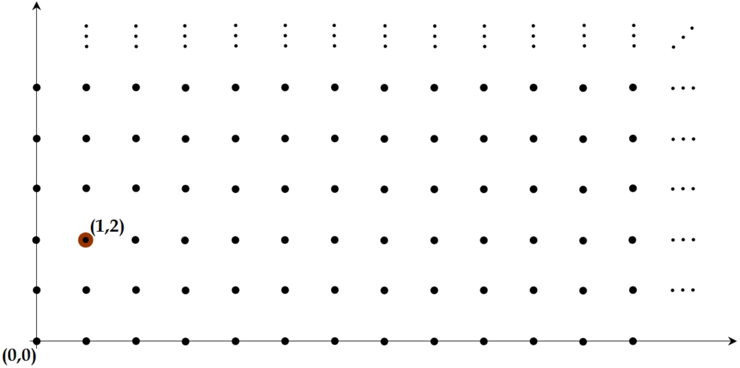

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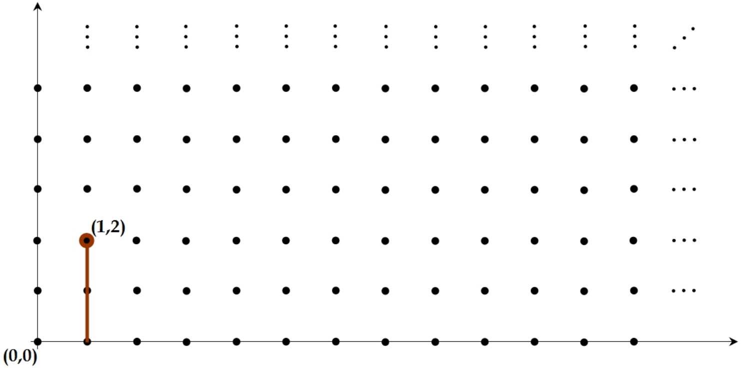

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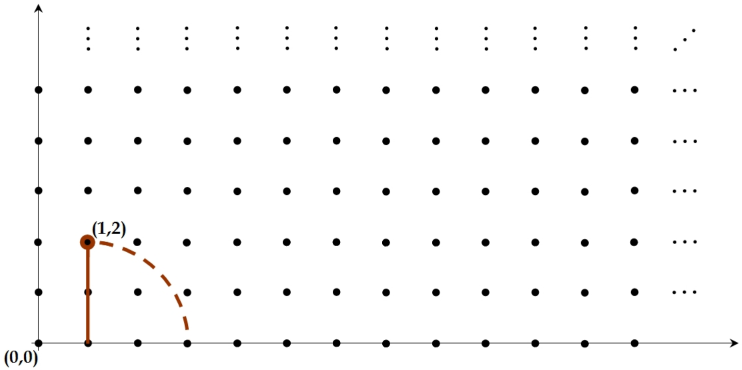

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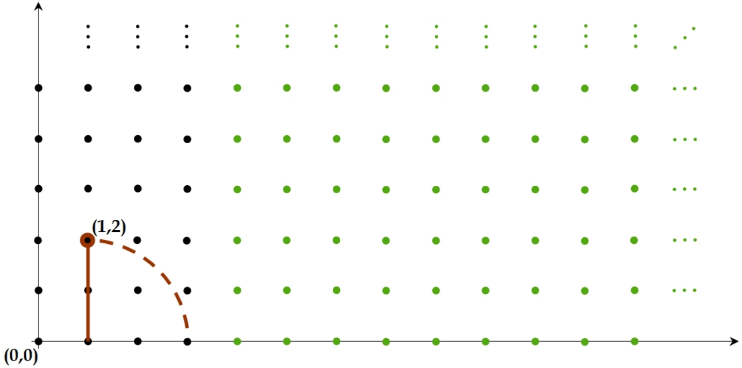

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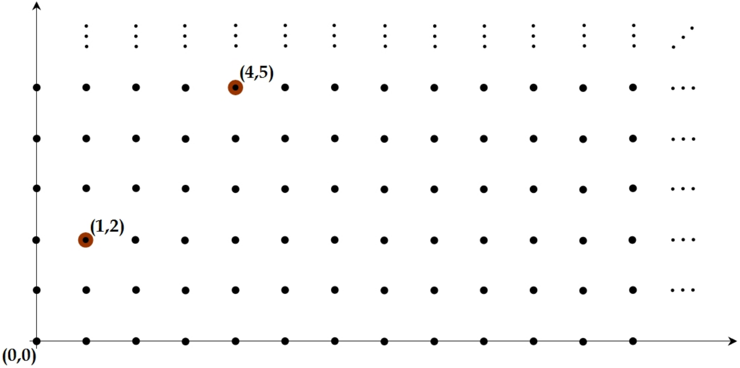

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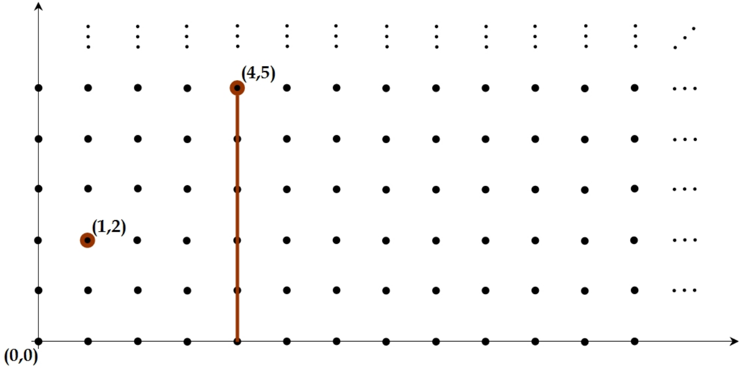

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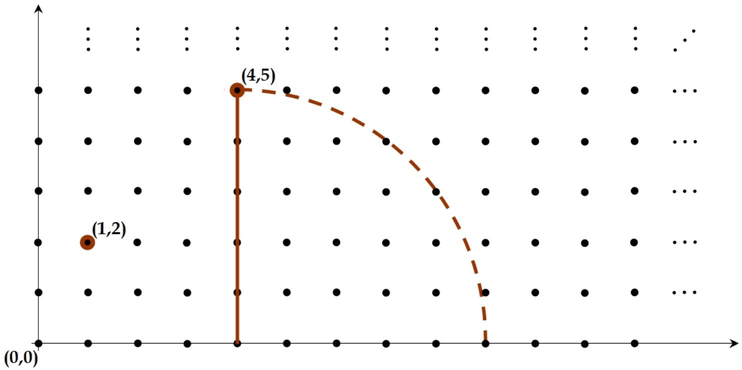

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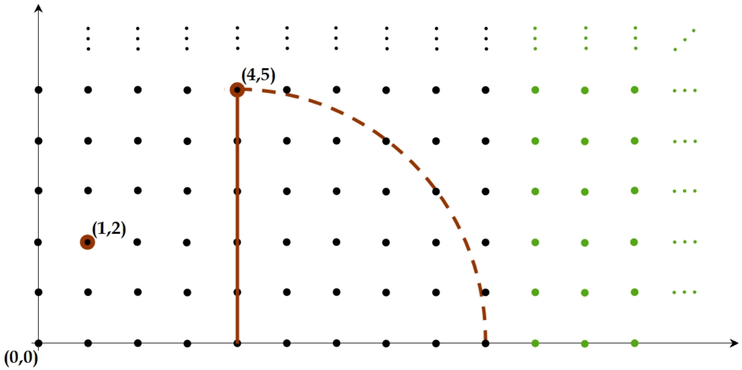

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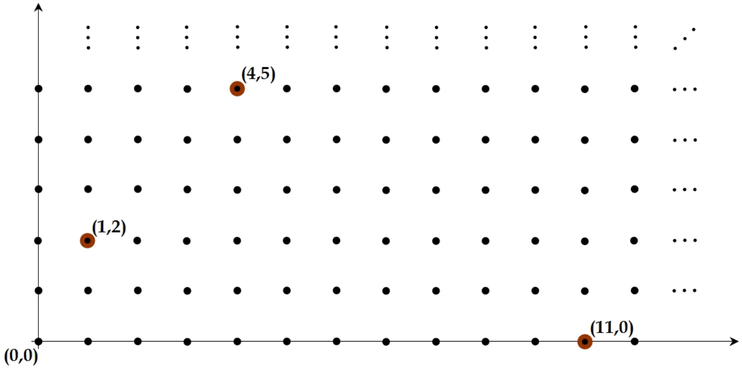

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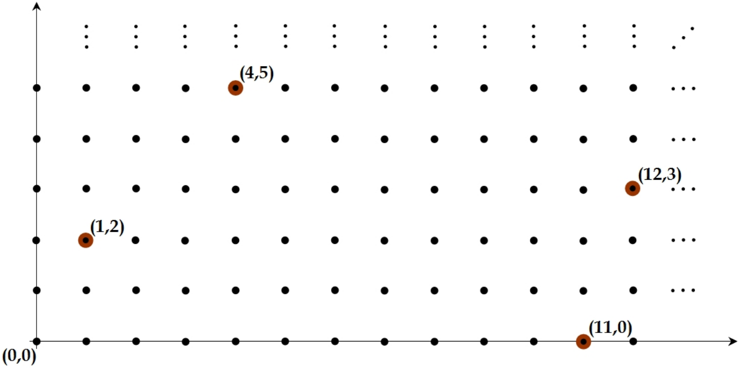

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# A (long) digression about ideal convergence - notations

- A sequence $(x_i)_{i\in\omega}$ of reals is $\mathcal{I}$-convergent to $x \in \mathbb{R}$ if $\{i \in \omega : |x_i - x| \geq \epsilon\} \in \mathcal{I}$ for every $\epsilon > 0$.
- A function $f : \mathbb{R} \to \mathbb{R}$ is a pointwise limit relatively to $\mathcal{I}$ of a sequence of functions $(f_i)_{i\in\omega}$ if $(f_i(x))_{i\in\omega}$ is $\mathcal{I}$-convergent to $f(x)$ for every $x \in \mathbb{R}$.
- For a family $\mathcal{F} \subset \mathbb{R}^{\mathbb{R}}$ by $LIM(\mathcal{F})$ we denote the family of all functions which can be represented as a pointwise limit of a sequence of functions from $\mathcal{F}$ (for instance, if $\mathcal{C}$ denotes the family of continuous functions then $LIM(\mathcal{C})$ is the first Baire class).
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**Theorem (Laczkovich and Recław, 2009)**

*Let $\mathcal{I}$ be a Borel ideal. TFAE:*

1. $\mathcal{I}$-$LIM(\mathcal{C}) = LIM(\mathcal{C})$;
2. $\mathcal{I}$ *is a weak P-ideal;*
3. $Fin \otimes Fin \not\leq_K \mathcal{I}$;
4. $Fin \otimes Fin \not\sqsubseteq \mathcal{I}$.

$\mathcal{I}$ is a weak P-ideal if for every $(X_i)_{i\in\omega} \subset \mathcal{I}$ there is $X \notin \mathcal{I}$ with $X \cap X_i$ finite for all $i$.
$\mathcal{I} \leq_K \mathcal{J}$ if there is $f\colon \bigcup\mathcal{J} \to \bigcup\mathcal{I}$ such that $f^{-1}[A] \in \mathcal{J}$ for all $A \in \mathcal{I}$. $\mathcal{I} \sqsubseteq \mathcal{J}$ if $f$ is a bijection.

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# Quasi-continuity

$f : \mathbb{R} \to \mathbb{R}$ is quasi-continuous ($f \in \mathcal{QC}$) if for every $\epsilon > 0$, $x_0 \in \mathbb{R}$ and open neighborhood $U \ni x_0$ there is a nonempty open $V \subset U$ such that $|f(x) - f(x_0)| < \epsilon$ for all $x \in V$.

Quasi-continuous are all continuous functions as well as all left-continuous (right-continuous) functions.

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$f : \mathbb{R} \to \mathbb{R}$ is quasi-continuous ($f \in \mathcal{QC}$) if for every $\epsilon > 0$, $x_0 \in \mathbb{R}$ and open neighborhood $U \ni x_0$ there is a nonempty open $V \subset U$ such that $|f(x) - f(x_0)| < \epsilon$ for all $x \in V$.

Quasi-continuous are all continuous functions as well as all left-continuous (right-continuous) functions.

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# Ideal convergence and quasi-continuity

**Theorem (Natkaniec and Szuca, preprint)**

*Let $\mathcal{I}$ be a Borel ideal. TFAE:*

1. $\mathcal{I}$-$LIM(\mathcal{QC}) = LIM(\mathcal{QC})$;
2. $\mathcal{I}$ *is weakly Ramsey;*

Following Laflamme we call $\mathcal{I}$ weakly Ramsey if every tree $T \subset [\omega]^{<\omega}$ with $\{n : s^\frown n \in T\}$ in the dual filter for all $s \in T$, contains an $\mathcal{I}$-positive branch.

**Question**

*Is there a counterpart of Fin $\otimes$ Fin for the above theorem?*

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# Another application of the ideal $\mathcal{K}$

**Theorem (K.)**

*Let $\mathcal{I}$ be any ideal. TFAE:*

1. $\mathcal{I}$ *is not weakly Ramsey;*
2. $\mathcal{K} \leq_K \mathcal{I}$;
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**Corollary**

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# More about weakly Ramsey ideals

Recall that $\mathcal{I}$ is locally selective, if every partition $(X_n)_{n\in\omega} \subset \mathcal{I}$ has an $\mathcal{I}$-positive selector.

$\mathcal{I}$ is weakly selective, if every partition $(X_n)_{n\in\omega}$ with at most one element not in $\mathcal{I}$ and such that $\bigcup_{m\geq n} X_m \notin \mathcal{I}$ for each $n$, has an $\mathcal{I}$-positive selector.

weakly selective $\Rightarrow$ locally selective

**Proposition (Essentially Grigorieff, 1971)**

*$\mathcal{I}$ is weakly Ramsey if and only if for every partition $(X_n)_{n\in\omega} \subset \mathcal{I}$, there exists a strictly increasing function $f : \omega \to \omega$, with $f[\omega] \notin \mathcal{I}$ and such that $f(n+1) \in \bigcup_{i>f(n)} X_i$ for each $n \in \omega$.*

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Thank you for your attention!