ORDERING FUNCTIONS

We aim at finding a "relevant" reduction notion for Borel functions from a zero-dimensional Polish space into another.

We give the requirements that make a quasi-order on functions relevant, for instance refining the order induced by the Baire hierarchy of functions, and generalising the embeddability order on Borel sets. We define several quasi-orders that we gather from different parts of the literature, and we discuss their adequacy as candidates to be relevant reduction notions with respect to the requirements.

We finally pick one candidate that we deem more interesting than the others, and study the properties of this quasi-order on continuous functions.

We generalise the Cantor-Bendixson analysis to continuous functions with countable range, prove that our quasi-order well orders continuous functions with compact domain, and give a general description of the relation between the Cantor-Bendixson layers.

If time allows it, we show how to obtain as a corollary that the closed subsets of a zero-dimensional Polish space are well-quasi-ordered by bicontinuous embeddability.