

def. A function $f : \mathbf{R} \rightarrow \mathbf{R}$ is weakly continuous (two sided) at $a \in \mathbf{R}$ if there are sequences $\{x_n\} \nearrow a$, $\{y_n\} \searrow a$ with $\lim_{n \rightarrow \infty} f(x_n) = \lim_{n \rightarrow \infty} f(y_n) = f(a)$.

thm. Every function has only countably many points of weak discontinuity and every countable set is a set of weak discontinuity points.

def. $a \in \mathbf{R}$ is a point of weak symmetric continuity if there are sequences $\{x_n\} \nearrow a$, $\{y_n\} \searrow a$ satisfying $\frac{x_n + y_n}{2} = a$ and $\lim_{n \rightarrow \infty} f(x_n) = \lim_{n \rightarrow \infty} f(y_n)$

The most interesting case is when we consider functions from \mathbf{R} to \mathbf{N}

thm. There is a function $f : \mathbf{R} \rightarrow \mathbf{N}$ with no points of weak symmetric continuity.

thm. Any subset of \mathbf{R} is a set points of weak symmetric continuity.

def. A function is T-weak symmetric at a if there are sequencies $\{x_n\} \nearrow a$, $\{y_n\} \searrow a$ satisfying

$$\frac{x_n + y_n}{2} = a \text{ and}$$

$$\forall_n f(x_n) = f(y_n) = f(a).$$

question Is every subset of \mathbf{R} a set of t-symmetric continuity for some $f : \mathbf{R} \rightarrow \mathbf{N}$?