Complete pairs of coanalytic sets

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Summary

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1. Borel, analytic and coanalytic sets
2. Definition of a complete pair
3. Basic examples of complete pairs
4. A complete pair in the space of continuous functions
5. A complete pair in the theory of automata

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Definition

X is a **Polish space** *if X is separable and completely metrizable.*

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The Borel sets $\mathcal{B}(X)$ *in a given topological space is the smallest σ–field containing all open sets of X.*

Definition

A set $A \subset X$ in a Polish space X is **analytic** *if there exists a Polish space Y and a Borel set $B \subset X \times Y$ such that*

$$A = \{x \in X : \exists y \in Y \ \langle x, y \rangle \in B\}.$$

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A set A in a topological space A is **Wadge reducible** *to a set B in a topological space Y if there exists a continuous mapping* $\phi : X \to Y$ *such that* $A = \phi^{-1}[B]$.

Definition

A disjoint pair A, B *in a topological space X is* **Wadge reducible** *to a disjoint C, D in a topological space Y, if there exists a continuous mapping* $\phi : X \to Y$ *such that* $A \leqslant_\phi C$ *and* $B \leqslant_\phi D$*, that is* $A = \phi^{-1}[C]$ *and* $B = \phi^{-1}[D]$.

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A disjoint pair of coanalytic sets C, D in a Polish space X is **complete**, *if for every disjoint pair of coanalytic sets A, B in the Cantor set the pair A, B is Wadge reducible to the pair C, D.*

The pair C, D represents all essential properties of pairs of coanalytic sets. For example, in the class of coanalytic sets there exists a pair A, B not separable by a Borel set. The same holds for all complete pairs.

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In order to prove that a given disjoint pair C, D of coanalytic sets is complete, it is enough to find a complete pair A, B and a reduction ϕ such that $A \leqslant_\phi C$ and $B \leqslant_\phi D$.

Definition

$T \subset \omega^{<\omega}$ is a **tree***, if T is closed with respect to initial segments, that is for every $s \in T$ and an initial segment $r \preceq s$ we have $r \in T$. A sequence $x \in \omega^{\omega}$ is a* **branch** *of T, if for every $n \in \omega$ we have $x|n \in T$.*

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Let Tr $\subset 2^{\omega^{<\omega}}$ *be the set of all trees. We define* WF *as the set of* **all well–founded trees** *and* UB *as the set of* **all trees with exactly one branch**.

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Every tree $T \in \mathrm{WF}$ *admits a natural* **rank** $\mathrm{rk}(T)$, *which is an ordinal below* ω_1. *Firstly we define inductively rank of* T *for every vertex of* T *and then define rank of* T *as the rank of* $\emptyset \in T$. *If* T *is not in* WF, *we define* $\mathrm{rk}(T)$ *as* ω_1.

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Let

$$V_0 = \{\langle S, T\rangle : S \in \mathrm{WF},\ \mathrm{rk}(S) < \mathrm{rk}(T)\}$$

and

$$V_1 = \{\langle S, T\rangle : T \in \mathrm{WF},\ \mathrm{rk}(T) \leqslant \mathrm{rk}(S)\}.$$

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We define Diff *as a subset of* $C([0,1])$ *consisting of all* **differentiable functions** *on the unit interval* $[0,1]$.

In 1936 S. Mazurkiewicz proved that the set Diff is an coanalytic non–Borel subset $C([0,1])$.

Definition

Let Diff_1 *be the set of all functions in* $C([0,1])$ *which are* **not differentiable in exactly one point** *of* $[0,1]$.

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Let $\mathcal{S}$ be the set of all **full binary trees** with vertices **labeled** by elements of the set $\{\exists, \forall\} \times \{0, 1\}$. Let $t \in \mathcal{S}$.

From a vertex of t one may go either right or left and the players $\exists$ and $\forall$ play a **game**, such that each of the players decides about a move from ‘his‘ vertices, that is from vertices labeled by $\exists$ and $\forall$ respectively.

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The player $\exists$ wins, if all vertices occurring in a given play, with except of finitely many, have label 0. **The player $\forall$ wins**, if all vertices occurring in a given play, with except of finitely many, has label 1.

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Let $W_{0,1}$ be the set of all trees in $\mathcal{S}$, such that the **player $\exists$ has a winning strategy** *and $W'_{0,1}$ be the set of all trees in $\mathcal{S}$, such that the* **player $\forall$ has a winning strategy**.

The pair $W_{0,1}$, $W'_{0,1}$ is a complete pair of coanalytic sets. Sz. Hummel proved in his Master Dissertation that the sets $W_{0,1}$, $W'_{0,1}$ are coanalytic and that the sets $W_{0,1}$, $W'_{0,1}$ are not separable by a Borel set.

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The set $W_{0,1}$ is an example of set accepted by a **non–deterministic tree automaton.** The automaton has the **states** $0, 1$ **and** T, works over the **alphabet** $\{\exists, \forall\} \times \{0, 1\}$ and has the following **transitions:**

$$i \stackrel{\langle\forall,j\rangle}{\longrightarrow} j, j,$$

$$i \stackrel{\langle\exists,j\rangle}{\longrightarrow} j, \mathrm{T}, \quad i \stackrel{\langle\exists,j\rangle}{\longrightarrow} \mathrm{T}, j$$

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The **rank** of the states 0 and T is 0 and the rank of the state 1 is 1. A tree $t \in \mathcal{S}$ is **recognized** by the automaton if there exists a run of the automaton such that on every branch x of t the $\limsup \rho(x(n))$ is even (in our case the only possible even rank is 0). The set $W'_{0,1}$ is accepted by a very similar automaton, such that the roles of $\exists$ and $\forall$ are swapped and at the same time the roles of 0 and 1 are swapped.

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An analogous definition gives sets $W_{i,k}, W'_{i,k}$ for larger sets of indices $\{i, \ldots, n\}$. One can prove, that the **complement of the set $W_{0,1}$ is not recognized** by an automaton with index $\{0, 1\}$ but is accepted by an automaton with index $\{1, 2\}$.

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It was proved by M. O. Rabin, that every two disjoint sets of trees A, B accepted by automata with indices $\{1,2\}$ is possible to separate by set C, which is simultaneously accepted by automata with indices $\{1,2\}$ and $\{0,1\}$.

For larger set of indices the question of separation remains open.

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