Finitely determined compact spaces

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Let $\mathcal{P}$ be a class of "small" compact spaces.

- A compact space K is $\mathcal{P}$-fibered if there exists a continuous map $f \colon K \to C$ such that C is metrizable and $f^{-1}(p) \in \mathcal{P}$ for every $p \in C$.
- A space X is $\mathcal{P}$-determined if there are a second countable space Σ and a usc multifunction $\Phi \colon \Sigma \to \mathcal{P}$ such that $X = \bigcup_{t \in \Sigma} \Phi(t)$.

Typically: $\mathcal{P}$ = metric compacta, finite sets, at most n-element sets.

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Some motivation

Theorem (Todorčević 1999)

Every Rosenthal compact is either 2-fibered or contains a copy of $A(\aleph_1)$.

Theorem (Tkachuk 1994)

Every Eberlein compact of weight $\leqslant 2^{\aleph_0}$ is metrizably determined.

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Problem (Fremlin)

Is it consistent with ZFC that every perfectly normal compact is 2-fibered?

Background

1. M. TKAČENKO, *$\mathcal{P}$-approximable compact spaces*, Comment. Math. Univ. Carolin. 32 (1991), no. 3, 583–595.
2. V. TKACHUK, *A glance at compact spaces which map "nicely" onto the metrizable ones*, Topology Proc. 19 (1994), 321–334.
3. W. KUBIŚ, O. OKUNEV, P. SZEPTYCKI, *On some classes of Lindelöf Sigma-spaces*, Topology Appl. 153 (2006), no. 14, 2574–2590.
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Basic properties

Theorem

A metrizably determined space does not contain uncountable free sequences.

Fact

A metrizably fibered compact is first countable and its images are Fréchet-Urysohn.

Example

Let $\mathcal{A}$ be a mad family on ω. Then $K = \mathrm{A}(\omega \cup \mathcal{A})$ is 2-determined, while it is not a continuous image of any metrizably fibered compact.

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Theorem (O. Okunev, P. Szeptycki & W.K.)

Every continuous image of a finitely fibered compact has a dense metrizable subspace.

Internal characterization

Proposition

Let X be a topological space and let $\mathcal{P}$ be a class of compacta. Then X is $\mathcal{P}$-determined iff

- *there are a family $\mathcal{C} \subseteq \mathcal{P}$ which covers X and a countable family $\mathcal{N}$ which forms a network for $\mathcal{C}$.*

That is:

$$(\forall\ C \in \mathcal{C})(\forall \text{ open } U \supseteq C)(\exists\ N \in \mathcal{N})\ \ C \subseteq N \subseteq U.$$

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Theorem (Tkachuk 1994)

Let $\mathcal{P}$ be a hereditary class of compacta. A compact space is $\mathcal{P}$-fibered iff it has a countable cover consisting of closed G_δ sets whose all maximal intersections are in $\mathcal{P}$.

Proof.

- Fix K and a suitable cover $\mathcal{N}$ consisting of closed G_δ sets.
- Fix a big enough χ and a countable $M \preceq H(\chi)$ so that $\mathcal{N} \in M$.
- Define $x \sim_M y$ iff $f(x) = f(y)$ for every $f \in \mathcal{C}(K) \cap M$.
- Let $q \colon K \to K/_M$ be the quotient map, where $K/_M = K/_{\sim_M}$.
- Given $x \in K$, we have that $q(x) = [x]_{\sim_M} \subseteq \mathcal{N}(x) \in \mathcal{P}$.

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Theorem

Let K be a compact space and let $\mathcal{P}$ be a hereditary class of compacta. Then K is $\mathcal{P}$-fibered iff for a sufficiently big regular cardinal χ, for every countable $M \preceq H(\chi)$ the canonical quotient

$$q\colon K \to K/_M$$

has fibers in $\mathcal{P}$.

Theorem (P. Szeptycki & W.K.)

A compact line is metrizably fibered iff it is embeddable into the lexicographic square

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A compact line determined by a special Aronszajn tree is metrizably determined.

Theorem

No Souslin line can be metrizably determined.

Proof.

- Let X be a Souslin line. Force with the associated Souslin tree.
- In the extension, $X \subseteq Y$, where Y is a line which has a point of uncountable character and X is dense in Y.
- Supposing X was metrizably determined, there is now a metrizably determined X' such that $X \subseteq X' \subseteq Y$.
- But X' has an uncountable free sequence!

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Trees

Let T be a tree. It is a locally compact space with the topology generated by intervals

$$(s,t] = \{x \in T \colon s < x \leqslant t\}.$$

Theorem (A. Mólto & W.K.)

Let T be a tree of cardinality $\leqslant 2^{\aleph_0}$. Then $\mathrm{A}(T)$ is 2-determined iff T is $\mathbb{R}$-embeddable.

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Corollary

There exists a scattered Rosenthal compact which is 3-determined, not 2-determined and not a continuous image of any first countable Rosenthal compact.

Proof.

- Take $T = \sigma\mathbb{Q}$, the Sierpiński tree of all bounded well ordered subsets of $\mathbb{Q}$.
- For each $t \in T$ partition the set of its immediate successors into infinite sets $L(t)$, $R(t)$.
- Add new elements $\ell(t)$, $r(t)$ just above t so that $\ell(t) < \ell$ for $\ell \in L(t)$ and $r(t) < r$ for $r \in R(t)$.
- The resulting tree is not $\mathbb{R}$-embeddable.
- Its Alexandrov compactification is Rosenthal compact.

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