

Pytkeev \aleph_0 -spaces

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Hejnice, 26 January 2014

Various networks in topological spaces

Definition

A family \mathcal{N} of subsets of a topological space X is called:

- a *network* if for any point $x \in X$ and neighborhood $O_x \subset X$ of x there is a set $N \in \mathcal{N}$ such that $x \in N \subset O_x$;
- a *k-network* if for any compact set $K \subset X$ and neighborhood $O_K \subset X$ of K there is a finite subfamily $\mathcal{F} \subset \mathcal{N}$ such that $K \subset \bigcup \mathcal{F} \subset O_K$;
- a *cs*-network* if for any point $x \in X$, neighborhood $O_x \subset X$ and convergent sequence $x_n \rightarrow x$ in X , there is a set $N \in \mathcal{N}$ such that $x \in N \subset O_x$ and $\{n \in \omega : x_n \in N\}$ is infinite;
- a *Pytkeev network* if for any point $x \in A$, neighborhood $O_x \subset X$, and set $A \subset X$ with $x \in \bar{A}$ there is a set $N \in \mathcal{N}$ such that $x \in N \subset O_x$ and $N \cap A$ is infinite if $x \in \bar{A} \setminus A$.

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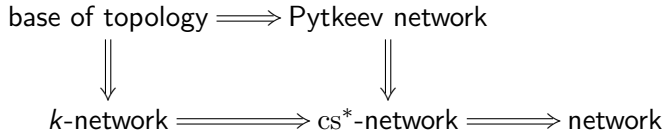
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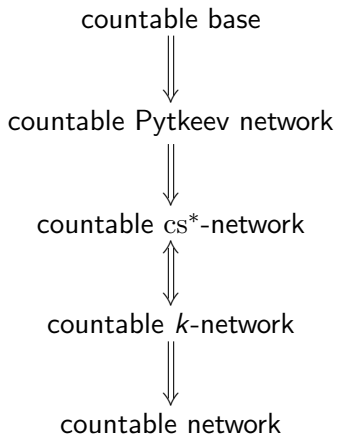
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Relations between various networks



Relations between various countable networks



Spaces with countable networks

Definition

A regular topological space X is called

- *cosmic* if X has a countable network;
- an \aleph_0 -space if X has a countable k -network;
- a *Pytkeev \aleph_0 -space* if X has a countable Pytkeev network.

\aleph_0 -spaces were introduced in 1966 by E. Michael.

They compose an important class of generalized metric spaces.

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Example

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For any ultrafilter $p \in \beta\mathbb{N}$ the space $X = \mathbb{N} \cup \{p\} \subset \beta\mathbb{N}$ is an \aleph_0 -space but not a Pytkeev \aleph_0 -space.

So, the class of Pytkeev \aleph_0 -spaces is properly contained in the class of \aleph_0 -spaces.

On the other hand, we have

Theorem

A sequential space is an \aleph_0 -space iff X is a Pytkeev \aleph_0 -space.

Question

What interesting can be said about the class of Pytkeev \aleph_0 -spaces?

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Countable fan tightness

Definition

A topological space X has *countable fan tightness* if for any sets $A_n \subset X$, $n \in \omega$, and a point $x \in \bigcap_{n \in \omega} \bar{A}_n$ there are finite sets $F_n \subset A_n$, $n \in \omega$, such that $x \in \text{cl}_X(\bigcup_{n \in \omega} F_n)$.

Theorem (B., 2013)

A topological space X is metrizable and separable if and only if X is a Pytkeev \aleph_0 -space with countable fan tightness.

So, second countable = Pytkeev \aleph_0 + countable fan tightness.

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Fact

A regular space X is cosmic if and only if X is a continuous image of a separable metric space.

Theorem (Michael, 1966)

For a topological space X the following conditions are equivalent:

- 1 X is a quotient image of a separable metric space;*
- 2 X is a sequential \aleph_0 -space;*
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Function spaces between \aleph_0 -spaces

For topological spaces X, Y by $C_k(X, Y)$ we denote the space of continuous functions from X to Y , endowed with the compact-open topology.

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Function spaces between Pytkeev \aleph_0 -spaces

Theorem (B., 2013)

For any \aleph_0 -space X and any Pytkeev \aleph_0 -space Y the function space $C_k(X, Y)$ is a Pytkeev \aleph_0 -space.

Corollary

For any \aleph_0 -space X the space $C_k(X)$ is a Pytkeev \aleph_0 -space and so is the space $C_k C_k(X)$.

Corollary

The countable Tychonoff product $\prod_{n \in \omega} X_n$ of Pytkeev \aleph_0 -spaces X_n , $n \in \omega$, is a Pytkeev \aleph_0 -space.

In fact, the class of Pytkeev \aleph_0 -spaces is closed under many countable operations over topological spaces.

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Inductive topologies

A topological space X carries the *inductive topology* with respect to a cover \mathcal{C} if the topology of X coincides with the strongest topology such that each identity inclusion $C \rightarrow X$, $C \in \mathcal{C}$, is continuous.

For example, the topological sum $\coprod_{\alpha \in A} X_\alpha$ of a disjoint family of topological spaces $(X_\alpha)_{\alpha \in A}$ carries the inductive topology with respect to the cover $\mathcal{C} = \{X_\alpha\}_{\alpha \in A}$.

Theorem

A regular topological space X is a Pytkeev \aleph_0 -space if X carries the inductive topology with respect to a countable cover \mathcal{C} by subsets which are Pytkeev \aleph_0 -spaces.

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For topological spaces X_α , $\alpha \in A$, their *box-product*

$$\prod_{\alpha \in A} X_\alpha$$

is the Cartesian product $\prod_{\alpha \in A} X_\alpha$ endowed with the topology generated by the base consisting of the products $\prod_{\alpha \in A} U_\alpha$ of open sets $U_\alpha \subset X_\alpha$.

Small box-products

A *pointed space* is a topological space X with a distinguished point $*_X \in X$.

For a family of pointed spaces X_α , $\alpha \in A$, their *small box-product*

$$\square_{\alpha \in A} X_\alpha = \left\{ (x_\alpha)_{\alpha \in A} \in \prod_{\alpha \in A} X_\alpha : \{\alpha \in A : x_\alpha \neq *_X\} \text{ is finite} \right\}$$

is a subspace of the box-product $\prod_{\alpha \in A} X_\alpha$.

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For a topological space X by its **hyperspace** $\exp(X)$ is the space of non-empty compact subsets endowed with the Vietoris topology.

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For any Pytkeev \aleph_0 -space X its hyperspace $\exp(X)$ is a Pytkeev \aleph_0 -space.

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Theorem

For any sequential \aleph_0 -space X the space $P_R(X)$ of probability Radon measures on X is a Pytkeev \aleph_0 -space.

Free objects over Pytkeev \aleph_0 -spaces

Let X be a Tychonoff space. Its *free abelian topological group* is any abelian topological group $A(X)$ algebraically generated by X so that any continuous map $f : X \rightarrow G$ to an abelian topological group G extends to a continuous homomorphism $\bar{f} : A(X) \rightarrow G$.

The *free locally convex space* is a locally convex space $L(X)$ having X as a Hamel basis such that any continuous map $f : X \rightarrow Y$ to a locally convex space Y extends to a continuous linear operator $\bar{f} : L(X) \rightarrow Y$.

It is known that for a k -space X the identity homomorphisms $A(X) \rightarrow L(X) \rightarrow C_k C_k(X)$ are topological embeddings.

Theorem (Leiderman, 2013)

For any sequential \aleph_0 -space X its free abelian topological group $A(X)$ and its free locally convex space $L(X)$ both are Pytkeev \aleph_0 -spaces.

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The class of Pytkeev \aleph_0 -spaces is a new class of generalized metric spaces, closed under taking subspaces, countable topological sums, countable inductive limits, countable Tychonoff products, countable box-products, countable inductive limits, function spaces C_k , hyperspaces, spaces of probability measures, and some free algebraic constructions.

T.Banakh, *Pytkeev \aleph_0 -spaces*, (2013);
<http://arxiv.org/abs/1311.1468>

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