# Pytkeev $\aleph_0$-spaces

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(Lviv and Kielce)

Hejnice, 26 January 2014

# Various networks in topological spaces

**Definition**

A family $\mathcal{N}$ of subsets of a topological space $X$ is called:

- *a network* if for any point $x \in X$ and neighborhood $O_x \subset X$ of $x$ there is a set $N \in \mathcal{N}$ such that $x \in N \subset O_x$;
- *a k-network* if for any compact set $K \subset X$ and neighborhood $O_K \subset X$ of $K$ there is a finite subfamily $\mathcal{F} \subset \mathcal{N}$ such that $K \subset \bigcup \mathcal{F} \subset \mathcal{N}$;
- a $\mathrm{cs}^*$*-network* if for any point $x \in X$, neighborhood $O_x \subset X$ and convergent sequence $x_n \to x$ in $X$, there is a set $N \in \mathcal{N}$ such that $x \in N \subset O_x$ and $\{n \in \omega : x_n \in N\}$ is infinite;
- a *Pytkeev network* if for any point $x \in A$, neighborhood $O_x \subset X$, and set $A \subset X$ with $x \in \bar{A}$ there is a set $N \in \mathcal{N}$ such that $x \subset N \subset \mathcal{N}$ and $N \cap A$ is infinite if $x \in \bar{A} \setminus A$.

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# Relations between various networks

$$\begin{array}{ccccc}
\text{base of topology} & \Longrightarrow & \text{Pytkeev network} & & \\
\Downarrow & & \Downarrow & & \\
k\text{-network} & \Longrightarrow & \mathrm{cs}^*\text{-network} & \Longrightarrow & \text{network}
\end{array}$$

# Relations between various countable networks

countable base

$\Downarrow$

countable Pytkeev network

$\Downarrow$

countable $\mathrm{cs}^*$-network

$\Updownarrow$

countable $k$-network

$\Downarrow$

countable network

# Spaces with countable networks

**Definition**

A regular topological space $X$ is called

- *cosmic* if $X$ has a countable network;
- an *$\aleph_0$-space* if $X$ has a countable $k$-network;
- a *Pytkeev $\aleph_0$-space* if $X$ has a countable Pytkeev network.

$\aleph_0$-spaces were introduced in 1966 by E.Michael.

They compose an important class of generalized metric spaces.

second countable $\Rightarrow$ Pytkeev $\aleph_0$-space $\Rightarrow$ $\aleph_0$-space $\Rightarrow$ cosmic

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# Example

**Example**

For any ultrafilter $p \in \beta\mathbb{N}$ the space $X = \mathbb{N} \cup \{p\} \subset \beta\mathbb{N}$ is an $\aleph_0$-space but not a Pytkeev $\aleph_0$-space.

So, the class of Pytkeev $\aleph_0$-spaces is properly contained in the class of $\aleph_0$-spaces.

On the other hand, we have

**Theorem**

*A sequential space is an $\aleph_0$-space iff $X$ is a Pytkeev $\aleph_0$-space.*

**Question**

What interesting can be said about the class of Pytkeev $\aleph_0$-spaces?

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# Countable fan tightness

**Definition**

A topological space $X$ has *countable fan tightness* if for any sets $A_n \subset X$, $n \in \omega$, and a point $x \in \bigcap_{n\in\omega} \bar{A}_n$ there are finite sets $F_n \subset A_n$, $n \in \omega$, such that $x \in \mathrm{cl}_X(\bigcup_{n\in\omega} F_n)$.

**Theorem (B., 2013)**

*A topological space $X$ is metrizable and separable if and only if $X$ is a Pytkeev $\aleph_0$-space with countable fan tightness.*

So, second countable = Pytkeev $\aleph_0$ + countable fan tightness.

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**Fact**

*A regular space $X$ is cosmic if and only if $X$ is a continuous image of a separable metric space.*

**Theorem (Michael, 1966)**

*For a topological space $X$ the following conditions are equivalent:*

1. *$X$ is a quotient image of a separable metric space;*
2. *$X$ is a sequential $\aleph_0$-space;*
3. *$X$ is a sequential Pytkeev $\aleph_0$-space.*

# Characterizations of cosmic and $\aleph_0$-spaces

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For topological spaces $X, Y$ by $C_k(X, Y)$ we denote the space of continuous functions from $X$ to $Y$, endowed with the compact-open topology.

**Theorem (Michael, 1966)**

*For any $\aleph_0$-spaces $X, Y$ the function space $C_k(X, Y)$ is an $\aleph_0$-space.*

# Function spaces between $\aleph_0$-spaces

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*For any $\aleph_0$-space $X$ and any Pytkeev $\aleph_0$-space $Y$ the function space $C_k(X, Y)$ is a Pytkeev $\aleph_0$-space.*

**Corollary**

*For any $\aleph_0$-space $X$ the space $C_k(X)$ is a Pytkeev $\aleph_0$-space and so is the space $C_k C_k(X)$.*

**Corollary**

*The countable Tychonoff product $\prod_{n\in\omega} X_n$ of Pytkeev $\aleph_0$-spaces $X_n$, $n \in \omega$, is a Pytkeev $\aleph_0$-space.*

In fact, the class of Pytkeev $\aleph_0$-spaces is closed under many countable operations over topological spaces.

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# Inductive topologies

A topological space $X$ carries the *inductive topology* with respect to a cover $\mathcal{C}$ if the topology of $X$ coincides with the strongest topology such that each identity inclusion $C \to X$, $C \in \mathcal{C}$, is continuous.

For example, the topological sum $\coprod_{\alpha \in X} X_\alpha$ of a disjoint family of topological spaces $(X_\alpha)_{\alpha \in A}$ carries the inductive topology with respect to the cover $\mathcal{C} = \{X_\alpha\}_{\alpha \in A}$.

**Theorem**

*A regular topological space $X$ is a Pytkeev $\aleph_0$-space if $X$ carries the inductive topology with respect to a countable cover $\mathcal{C}$ by subsets which are Pytkeev $\aleph_0$-spaces.*

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# Box-products

For topological spaces $X_\alpha$, $\alpha \in A$, their *box-product*

$$\square_{\alpha \in A} X_\alpha$$

is the Cartesian product $\prod_{\alpha \in A} X_\alpha$ endowed with the topology generated by the base consisting of the products $\prod_{\alpha \in A} U_\alpha$ of open sets $U_\alpha \subset X_\alpha$.

# Small box-products

A *pointed space* is a topological space $X$ with a distinguished point $*_X \in X$.

For a family of pointed spaces $X_\alpha$, $\alpha \in A$, their *small box-product*

$$\boxdot_{\alpha\in A} X_\alpha = \{(x_\alpha)_{\alpha\in A} \in \square_{\alpha\in A} X_\alpha : \{\alpha \in A : x_\alpha \neq *_{X_\alpha}\} \text{ is finite}\}$$

is a subspace of the box-product $\square_{\alpha\in A} X_\alpha$.

**Theorem**

*For any sequence $X_n$, $n \in \omega$, of pointed Pytkeev $\aleph_0$-spaces their small box-product $\boxdot_{n\in\omega} X_n$ is a Pytkeev $\aleph_0$-space.*

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For a topological space $X$ by its hyperspace $\exp(X)$ is the space of non-empty compact subsets endowed with the Vietoris topology.

**Theorem**

*For any Pytkeev $\aleph_0$-space $X$ its hyperspace $\exp(X)$ is a Pytkeev $\aleph_0$-space.*

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**Theorem**

*For any sequential $\aleph_0$-space $X$ the space $P_R(X)$ of probability Radon measures on $X$ is a Pytkeev $\aleph_0$-space.*

Let $X$ be a Tychonoff space. Its *free abelian topological group* is any abelian topoloigical group $A(X)$ algebraically generated by $X$ so that any continuous map $f : X \to G$ to an abelian topological group $G$ exends to a continuous homomorphism $\bar{f} : A(X) \to G$.

The *free locally convex space* is a locally convex space $L(X)$ having $X$ as a Hamel basis such that any continuous map $f : X \to Y$ to a locally convex space $Y$ extends to a continuous linear operator $\bar{f} : L(X) \to Y$.

It is known that for a $k$-space $X$ the identity homomorphisms $A(X) \to L(X) \to C_k C_k(X)$ are topological embeddings.

**Theorem (Leiderman, 2013)**

*For any sequential $\aleph_0$-space $X$ its free abelian topological group $A(X)$ and its free locally convex space $L(X)$ both are Pytkeev $\aleph_0$-spaces.*

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# Conclusion

The class of Pytkeev $\aleph_0$-spaces is a new class of generalized metric spaces, closed under taking subspaces, countable topological sums, countable inductive limits, countable Tychonoff products, countable box-products, countable inductive limits, function spaces $C_k$, hyperspaces, spaces of probability measures, and some free algebraic constructions.

# Reference

T.Banakh, *Pytkeev $\aleph_0$-spaces*, (2013);
http://arxiv.org/abs/1311.1468

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Thanks to:

- Arkady Leiderman who posed a proper question (about function space $C_k(\mathbb{Q})$) in a proper place (Warsaw) in a proper time (13 October 2013);
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