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Large free subgroups of automorphisms group of ultrahomogeneous spaces

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large free groups in groups of automorphism

groups of automorphisms

Let A be a countable structure (in fact, we should write $\mathcal{A} = (A, \mathcal{F}, \mathcal{R}, \mathcal{C})$). By $\text{Aut}(A)$ we denote the group of automorphisms of A.

general problem

Detect those countable structures A, whose groups of auomorphisms $\text{Aut}(A)$ contains a large free group.

Macperson (1986)

If A is ω-categorical, then $\text{Aut}(A)$ contains a dense free subgroup of ω generators.
Automorphism group of a random graph contains a dense free subgroup of 2 generators.

Melles and Shelah (1994)

If A is a saturated model of a complete theory T with $|A| = \lambda > |T|$, then $\text{Aut}(A)$ has a dense free subgroup of of cardinality 2^λ.

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We say that a countable structure A is *ultrahomogeneous*, if each isomorphism between finitely generated substructures of A can be extended to an automorphism of A.

examples

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Detect those countable ultrahomogeneous structures A such that there exists a family $\mathcal{H} \subset \mathrm{Aut}(A)$ of $\mathfrak{c}$-many free generators.
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free generators

words

Let $y_1, y_2, \ldots$ be a set of letters, $m, k \geq 1$, $r_1, \ldots., r_k \in \{1, \ldots, m\}$ be such that $r_i \neq r_{i+1}$ for $i \in \{1, \ldots, k-1\}$, and $n_1, \ldots, n_k \in \mathbb{Z} \setminus \{0\}$. Then

$$w(y_1, \ldots, y_m) = y_{r_1}^{n_1} y_{r_2}^{n_2} \ldots y_{r_k}^{n_k}$$

is called a *word of length* n, where $n = |n_1| + \ldots + |n_k|$.

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A family $\mathcal{H} \subset Aut(A)$ is a family of free generators if for every word $w(y_1, \ldots., y_m)$ and every distinct $f_1, \ldots, f_m \in \mathcal{H}$, the function

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positive example

Let $A = \omega$. Then $\mathrm{Aut}(A) = S^\infty$ - the group of all bijections of ω.
S^∞ is $\mathfrak{c}$-large.

negative example

Let $A = (\omega, \{R_n : n \in \omega\})$, where R_n are unary relations such that

$$x \in R_n \text{ iff } x \in \{2n, 2n+1\}.$$

Then for every $f \in \mathrm{Aut}(A)$, $f \circ f = \mathrm{id}$, so $\mathrm{Aut}(A)$ does not contain any nonempty family of free generators.

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the Rasiowa-Sikorski Lemma

filters,dense sets

Let $(P, \leq)$ be a partially ordered set (poset).
We say that a set $G \subset P$ is a *filter*, if:
– for every $p, q \in P$, if $p \leq q$ and $p \in G$, then $q \in G$;
– for every $p_1, p_2 \in G$, there is $q \in G$ such that $q \leq p_i$ for $i = 1, 2$.
We say that a set $D \subset P$ is *dense*, if:
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Let $(P, \leq)$ be a poset with ccc (in particular, countable) and $\{D_n : n \in \omega\}$ be a family of dense subsets of P.
Then there is a filter $G \subset P$ (called *a generic filter*) such that for every $n \in \omega$, $G \cap D_n \neq \emptyset$.

our idea

We will construct a countable poset $(\mathbb{P}, \leq)$, and a family of dense sets $\{D_n : n \in \omega\}$ such that the generic filter G will generate the family of $\mathfrak{c}$ many free generators.

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By $\mathbb{P}$ we denote the set of pairs (n, p), such that

- $n \in \omega$;
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The set $\mathbb{P}$ is ordered in the following way: $(n, p) \leq (k, q)$ iff

- $n \geq k$;
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$(\mathbb{P}, \leq)$ is countable.

partial automorphisms generated by a filter

Let G be a filter on $(\mathbb{P}, \leq)$ and $\alpha \in \{0,1\}^\omega$. Then

$$g(\alpha) = \bigcup \{p(\alpha|_n) : (p, n) \in G\}$$

is a partial automorphism. Hence $\{g(\alpha) : \alpha \in \{0,1\}^\omega\} \subset \mathrm{Part}(A)$.

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sets D_k

For every $k \in A$, let

$$D_k = \{(p, n) \in \mathbb{P} : \forall_{s \in \{0,1\}^n}\ k \in \mathrm{dom}(p(s)) \cap \mathrm{rng}(p(s))\}$$

sets $D_w^{s_1,\dots,s_m}$

For every word $w(y_1, \dots, y_m)$, every $k \in \mathbb{N}$ and every pairwise distinct $s_1, \dots, s_m \in \{0,1\}^k$, define

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sets D_k and $D_w^{s_1,\dots,s_m}$ are "good"

Assume that G is a filter on $\mathbb{P}$ such that $G \cap D_k \neq \emptyset$ and $G \cap D_w^{s_1,\dots,s_m} \neq \emptyset$ for all $k, w, s_1, \dots, s_m$.
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Let $k \in A$. Let $(p, n) \in G \cap D_k$. Then for any $\alpha \in \{0,1\}^\omega$,

$$k \in \mathrm{dom}(p(\alpha|_n)) \cap \mathrm{rng}(p(\alpha|_n)) \subset \mathrm{dom}(g(\alpha)) \cap \mathrm{rng}(g(\alpha)).$$

Let $w(y_1, \dots, y_m)$ be a word and $\alpha_1, \dots, \alpha_m \in \{0,1\}^\omega$ be distinct. Then there is $k \in \omega$ such that $\alpha_1|_k, \dots., \alpha_m|_k$ are distinct.
Let $(n, p) \in G \cap D_w^{\alpha_1|_k,\dots,\alpha_m|_k}$. Then there is $x_0 \in A$ such that

$$w(p(\alpha_1|_n), \dots, p(\alpha_m|_n))(x_0) \neq x_0.$$

In particular,

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sets D_k and $D_w^{s_1,\dots,s_m}$ are "good"

Assume that G is a filter on $\mathbb{P}$ such that $G \cap D_k \neq \emptyset$ and $G \cap D_w^{s_1,\dots,s_m} \neq \emptyset$ for all $k, w, s_1, \dots, s_m$.
Then $\{g(\alpha) : \alpha \in \{0,1\}^\omega\}$ is a family of $\mathfrak{c}$ many free generators.

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Let $k \in A$. Let $(p, n) \in G \cap D_k$. Then for any $\alpha \in \{0,1\}^\omega$,

$$k \in \mathrm{dom}(p(\alpha|_n)) \cap \mathrm{rng}(p(\alpha|_n)) \subset \mathrm{dom}(g(\alpha)) \cap \mathrm{rng}(g(\alpha)).$$

Let $w(y_1, \dots, y_m)$ be a word and $\alpha_1, \dots, \alpha_m \in \{0,1\}^\omega$ be distinct. Then there is $k \in \omega$ such that $\alpha_1|_k, \dots, \alpha_m|_k$ are distinct.
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what should be assumed about A?

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What should be assumed about A, sets D_k and $D_w^{s_1,\dots,s_m}$ are dense in $\mathbb{P}$?

denseness of D_k

Assume that A is such that each finitely generated substructure is finite. Then for every $k \in \mathbb{N}$, the set D_k is dense in $\mathbb{P}$.

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sets $D_w^{s_1,\dots,s_m}$ - $(x_0,\dots,x_n)$-functions

$(x_0,\dots,x_n)$-function

Let $x_0,\dots,x_n$ be distinct elements. We say that a function g is an $(x_0,\dots,x_n)$-*function*, if there are integers $0 \le a_1 < b_1 < a_2 < b_2 < \dots < a_k < b_k \le n$ such that for every $r = 1,\dots,k$,

$$g(x_i) = x_{i+1} \text{ for every } i = a_r,\dots,b_r - 1$$

or

$$g(x_i) = x_{i-1} \text{ for every } i = a_{r+1},\dots,b_r$$

and dom(g) contains exactly those $x_i's$ which appear in the above condition.

$(x_0,\dots,x_n)$-functions are good

For every nonempty word $w(y_1,\dots,y_m)$ of the length n, and distinct $x_0,\dots,x_n$, there exist $(x_0,\dots,x_n)$-functions $g_1,\dots,g_m$ such that $w(g_1,\dots,g_m)(x_0) = x_n$.

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(∗) For any finitely generated substructures $B_1, B_2 \subset A$ and any $m \in \mathbb{N}$, there exist pairwise distinct $x_0, \dots, x_n \in A \setminus (B_1 \cup B_2)$ such that for any embedding $f : B_1 \to B_2$, and for any $(x_0, \dots, x_n)$-function g, there exists an embedding $f_g : \mathrm{gen}(B_1 \cup \mathrm{dom}(g)) \to A$ such that $f, g \subset f_g$.

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Assume that A satisfies (∗) and each finitely generated substructure of A is finite. Then each set $D_w^{s_1,\dots,s_m}$ is dense in $\mathbb{P}$.

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Assume that A satisfies (∗) and each finitely generated substructure of A is finite. Then $\mathrm{Aut}(A)$ is $\mathfrak{c}$-large.

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corollaries

ω

The structure ω satisfies $(*)$ and every finitely generated substructure is finite.

S^∞ group

The group S^∞ of all bijections of ω is $\mathfrak{c}$-large.

$(\mathbb{Q}, \leq)$

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rational Urysohn space

A *rational Urysohn space* is a countable metric space U with rational distances, such that every finite metric space with rational distances has an isometric copy in U.
A rational metric space satisfies $(*)$ and finite substructures are finite.

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The group of all isometries of a rational Urysohn space U is $\mathfrak{c}$-large.

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A *random graph* $\mathbb{G}$ is a countable graph such that for every finite $X, Y \subset \mathbb{G}$, there is a vertex with edges going to each vertex from X, and no edge going to a vertex of Y.
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more complicated problem

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Let $\mathcal{F} \subset \mathrm{Aut}(A)$ be a family of free generators. Does there exists a family $\mathcal{H}$ of cardinality $\mathfrak{c}$ such that $\mathcal{F} \cup \mathcal{H}$ is a family of free generators?

theorem, Głąb, S. (2013)

Let $\mathcal{F} \subset S^\infty$ be a countable family of free generators. Then there is a family $\mathcal{H} \subset S^\infty$ of cardinality $\mathfrak{c}$ such that $\mathcal{F} \cup \mathcal{H}$ is a family of free generators.

idea of a proof

(here $A = \omega$)
For every word $w(y_1, ..., y_{m+l})$, every $s_1, ..., s_m \in \{0,1\}^k$ and every $f_1, ..., f_l \in \mathcal{F}$, the set

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Assume $MA(\mathfrak{c})$. Then for every family $\mathcal{F} \subset S^\infty$ of less than $\mathfrak{c}$ many free generators, there exists a family $\mathcal{H} \subset S^\infty$ of cardinality $\mathfrak{c}$ such that $\mathcal{H} \cup \mathcal{F}$ is a family of free generators.

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Thank you for your attention

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