Winter School in Abstract Analysis

Another cardinal invariant for ideals on ω

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Joint work with M. Hrušák

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Definition.

- Given an ideal $\mathscr{I}$ on ω, an ultrafilter $\mathcal{U}$ is an $\mathscr{I}$-ultrafilter if and only if for any $f \in \omega^\omega$ there exists $A \in \mathcal{U}$ such that $f[A] \in \mathscr{I}$.
- $\mathcal{U}$ is a weak $\mathscr{I}$-ultrafilter if for any $f \in \omega^\omega$ finite to one there exists $A \in \mathcal{U}$ such that $f[A] \in \mathscr{I}$.

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Proposition.

- An ultrafilter $\mathcal{U}$ is a p-point if and only if $\mathcal{U}$ is a $Fin \times Fin$-ultrafilter.
- An ultrafilter $\mathcal{U}$ is q-point if and only if $\mathcal{U}$ is a weak $\mathcal{ED}_{fin}$-ultrafilter.

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Some comments about parametrized diamond principles.

- Parametrized diamiond-like principles were introduce by M. Džamonja, M. Hrušák and J. T. Moore.
- This principles are a weakening of the Jensen's diamond principle $\diamondsuit$ that are compatible with the negation of CH.
- For every **Borel** cardinal invariants there is a correspondent parametrized diamond-like principle.
- For many **non-Borel** cardinal invariants there is a **Borel** cardinal invariant which implies the former to be $\aleph_1$. For example:
  1. $\diamondsuit(\mathfrak{r})$ implies $\mathfrak{u} = \aleph_1$.
  2. $\diamondsuit(\mathfrak{b})$ implies $\mathfrak{a} = \aleph_1$.

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Theorem.

If $\mathfrak{d} = \omega_1$, then there is a q-point (weak $\mathcal{ED}_{fin}$-ultrafilters).

Theorem(M. Dzamonja, M. Hrušák, J. T. Moore)

$\diamondsuit(\mathfrak{r})$ implies the existence of p-points ($Fin \times Fin$-ultrafilters).

Given a Borel ideal $\mathscr{I}$, does there exists a cardinal invariant $\mathfrak{z}$ such that $\diamondsuit(\mathfrak{z})$ implies the existence of (weak) $\mathscr{I}$-ultrafilters?

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Given a Borel ideal $\mathscr{I}$, does there exists a cardinal invariant $\mathfrak{z}$ such that $\diamondsuit(\mathfrak{z})$ implies the existence of (weak) $\mathscr{I}$-ultrafilters?

Definition.

Let $\mathscr{I}$ be a tall Borel ideal. Define a cardinal invariant $\mathfrak{z}(\mathscr{I})$ as follows:

$$\mathfrak{z}(\mathscr{I}) = \min\{|\mathcal{D}| : (\mathcal{D} \subseteq [\omega]^\omega)(\forall f \in \omega^\omega)(\exists A \in \mathcal{D})(f[A] \in \mathscr{I})\}$$

Similarly, define $\mathfrak{z}_{fin}(\mathscr{I})$ as:

$$\mathfrak{z}_{fin}(\mathscr{I}) = \min\{|\mathcal{D}| : (\mathcal{D} \subseteq [\omega]^\omega)(\forall f \in \omega^\omega \text{ finite to one})(\exists A \in \mathcal{D})(f[A] \in \mathscr{I})\}$$

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Let $\mathscr{I}$ be a tall Borel ideal, then the diamond-like principles associated to $\mathfrak{z}(\mathscr{I})$ and $\mathfrak{z}_{fin}(\mathscr{I})$ are the following:

$\diamondsuit(\mathfrak{z}(\mathscr{I}))$

For all Borel function $F : 2^{<\omega_1} \to \omega^\omega$ there is a function $g : \omega_1 \to [\omega]^\omega$ such that for any $f \in 2^{\omega_1}$, the set $\{\alpha \in \omega_1 : F(f \restriction \alpha)[g(\alpha)] \in \mathscr{I}\}$ is stationary.

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For all Borel function $F : 2^{<\omega_1} \to \omega^\omega$ with range the set of finite to one functions, there is a function $g : \omega_1 \to [\omega]^\omega$ such that for any $f \in 2^{\omega_1}$, the set $\{\alpha \in \omega_1 : F(f \restriction \alpha)[g(\alpha)] \in \mathscr{I}\}$ is stationary.

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Proposition.

Let $\mathscr{I}$ be a Borel tall ideal. Then:

- $\Diamond(\mathfrak{z}(\mathscr{I}))$ implies the existence of $\mathscr{I}$-ultrafilters.
- $\Diamond(\mathfrak{z}_{fin}(\mathscr{I}))$ implies the existence of weak $\mathscr{I}$-ultrafilters.

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Remark.

Ramsey ultrafilters are $\mathscr{I}$-ultrafilters for all Borel ideal $\mathscr{I}$. In particular $\mathfrak{z}(\mathscr{I}) \leq$ the minimum character of a Ramsey ultrafilter (provided they exist).

Proposition.

It is consistent that for all Borel tall ideal $\mathscr{I}$, $\mathfrak{z}(\mathscr{I}) < \mathfrak{c}$.

A Ramsey ultrafilter $\mathcal{U}$ is an $\mathscr{I}$-ultrafilter for all analytic ideal $\mathscr{I}$ and that in the Sacks model there are Ramsey ultrafilters of small character.

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Lemma.

For any Borel ideal $\mathscr{I}$, $\mathfrak{z}(\mathscr{I}) \leq \max\{\mathfrak{z}_{fin}(\mathscr{I}), \mathfrak{r}_\sigma\}$.

Proposition.

For any tall meager ideal $\mathscr{I}$ we have $\mathfrak{z}_{fin}(\mathscr{I}) \geq \min\{\mathfrak{d}, \mathfrak{r}\}$.

Proposition.

If $\mathscr{I}$ is an ideal and there exists a coloring $\varphi : [\omega]^n \to k$ such that all φ-homogeneous sets belong to the ideal $\mathscr{I}$, then $\mathfrak{z}(\mathscr{I}) \leq \max\{\mathfrak{d}, \mathfrak{r}_\sigma\}$.

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Proposition.

$\mathfrak{z}_{fin}(\mathscr{I}) \leq \mathfrak{d}$ for all analytic p-ideal on ω.

Theorem.

It is consistent that for all analytic tall p-ideal $\mathscr{I}$ $\mathfrak{z}_{fin}(\mathscr{I}) < \mathfrak{d}$.

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Questions.

- Is $\mathfrak{z}_{fin}(\mathscr{I}) = \min\{\mathfrak{d}, \mathfrak{r}\}$ for all analytic p-ideal? This holds for the ideal $\mathcal{Z}$.
- Is it consistent that there exist a Borel ideal $\mathscr{I}$ such that $\mathfrak{z}(\mathscr{I}) > \max\{\mathfrak{d}, \mathfrak{r}_\sigma\}$?

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Theorem(Vojtáš).

An ultrafilter $\mathcal{U}$ is rapid if and only if it has non-empty intersection with any tall summable ideal.

Theorem (J. Flašková).

There is a family $\mathcal{D}$ of tall summable ideals of cardinality $\mathfrak{d}$ such that for any ultrafilter $\mathcal{U}$, $\mathcal{U}$ is rapid if and only if $\mathscr{I} \in \mathcal{D}$ $\mathcal{U} \cap \mathscr{I}$ is not empty.

Question (J. Flašková).

What is the minimal size of a family $\mathcal{D}$ of tall summable ideals such that rapid ultrafilters can be characterized as those ultrafilters on the natural numbers which have a nonempty intersection with all ideals in the family $\mathcal{D}$?

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Proposition(*).

For any family of tall summable ideals $\mathcal{D}$ with $|\mathcal{D}| < \mathfrak{d}$, there is an ultrafilter $\mathcal{U}$ which meets all ideal $\mathscr{I} \in \mathcal{D}$, but $\mathcal{U}$ is not a rapid ultrafilter.

Corollary.

$\mathfrak{d}$ is equal to the minimum cardinality of a nonempty family $\mathcal{D}$ of tall summable ideals such that for any ultrafilter $\mathcal{U}$, $\mathcal{U}$ is rapid if and only if $\mathcal{U}$ meets all ideals $\mathscr{I} \in \mathcal{D}$.

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A stronger version of Proposition(*) is consistent.

Definition.

Let $\mathscr{I}$ be an ideal on ω. For an ultrafilter $\mathcal{U}$, let's say that $\mathcal{U}$ is an $(\mathscr{I}, p)$-point if $\mathcal{U}$ is a p-point and also an $\mathscr{I}$-ultrafilter.

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Theorem

Rational Perfect Set Forcing preserves $(\mathscr{I}, p)$-points for any analytic p-ideal $\mathscr{I}$.

Theorem

Let $\mathscr{I}$ be an F_σ p-ideal and let $\mathcal{U}$ be an $(\mathscr{I}, p)$-point. Let $\mathbb{P}_\alpha = \langle \mathbb{P}_\beta, \dot{\mathbb{Q}}_\beta : \beta < \alpha \rangle$ be a CSI of proper forcing notions such that for all $\beta < \alpha$, $\mathbb{P}_\beta \Vdash \dot{\mathbb{Q}}_\beta$ *preserves* $(\mathscr{I}, p)$*-points*. Then $\mathbb{P}_\alpha$ preserves $(\mathscr{I}, p)$-points.

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Putting this two theorems together we obtain.

Theorem.

In the Rational Perfect Set Forcing model, given any family $\mathcal{D}$ of tall summable ideals with $|\mathcal{D}| < \mathfrak{d}$, there is an ultrafilter $\mathcal{U}$ such that is an $\mathscr{I}$-ultrafilter for all $\mathscr{I} \in \mathcal{D}$, but there is no rapid ultrafilter.

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Thank you for your attention!