Marek Bienias

Some properties of compact preserving functions

Joint results with Taras Banakh, Artur Bartoszewicz and Szymon Głąb

Notion

Definition

A function $f : X \to Y$ between topological spaces is called *compact-preserving*, provided the set $f(K) \subseteq Y$ is compact for any compact set $K \subseteq X$

Example

- any continuous function;
- any function with finite range.

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A function $f : \mathbb{R} \to \mathbb{R}$ is continuous iff f is compact preserving and has the Darboux property (i.e. maps connected sets to connected sets).

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Fréchet spaces

We recall that a topological space X is

- *Fréchet* if for each $A \subset X$ and $a \in \bar{A}$ there is $\{a_n\}_{n\in\omega} \subset A$ that converges to a;
- *strong Fréchet* if for any decreasing sequence $\{A_n\}_{n\in\omega} \subseteq X$ and any $a \in \bigcap_{n\in\omega} \bar{A}_n$ there is a sequence $a_n \in A_n$, $n \in \omega$, that converges to a.

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Let $f : X \to Y$ and $x \in X$.

Definition

$$f[x] = \{y \in Y : x \in \mathrm{cl}_X(f^{-1}(y))\}$$

$$= \bigcap \{f(O_x) : O_x \text{ is a neighborhood of } x \text{ in } X\},$$

$f[x]$ can be interpreted as the oscillation of f at x.
If f is continuous at x and Y is a T_1-space, then the set $f[x] = \{f(x)\}$.

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$X-$ Fréchet space and $Y-$ Hausdorff space.

Theorem 1

Let $f : X \to Y$ be compact-preserving. Then

$$\forall_{x \in X} \forall_{O_{f(x)}} \exists_{O_x} f(O_x) \subset f[x] \cup O_{f(x)}$$

If X is strong Fréchet, then $f[x] \setminus O_{f(x)}$ is finite.

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Sketch of the proof

Step 1 Suppose on the contrary that there is x_0 without this property;

Step 2 Denote $A = f^{-1}(Y \setminus (f[x_0] \cup O_{f(x_0)}))$ and notice than $x_0 \in \bar{A}$;

Step 3 Take a sequence $\{x_n\} \subseteq A$ s.t. $x_n \to x_0$;

Step 4 Observe that $\{f(x_n)\}$ is one-to-one from some place;

Step 5 Notice that a set $K = \{f(x_0)\} \cup \{f(x_n)\}_{n\in\omega}$ is compact and infinite;

Step 6 Throw out a non-isolated point y_0 from K, $K \setminus \{y_0\}$ is not compact;

Step 7 Observe that $y_0 \neq f(x_0)$;

Step 8 But $K \setminus \{y_0\} = f(S)$ where $S = \{x_0\} \cup \{x_n\}_{n\in\omega} \setminus f^{-1}(y_0)$ is compact.
Contradiction!

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Corollary

Function $f: X \to Y$ is compact-preserving if (and only if)

$$\forall_{x \in X} \exists_{K_x \subseteq Y} \forall_{O_{f(x)}} \exists_{O_x} f(O_x) \subset K_x \cup O_{f(x)}$$

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Well known

A function $f : \mathbb{R} \to \mathbb{R}$ is continuous if and only if f is compact preserving and has the Darboux property (i.e. maps connected sets to connected sets).

$X-$ locally connected strong Fréchet space and $Y-$ Hausdorff space.

Generalization

A function $f : X \to Y$ is continuous if and only if f is compact-preserving and has the Darboux property.

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Let $f : X \to Y$. A sequence $\{x_n\}_{n\in\omega} \subseteq X$ is called

- *injective* if $x_n \neq x_m$ for $n \neq m$;
- *f-injective* if $f(x_n) \neq f(x_m)$ for $n \neq m$.

Observation

For any compact-preserving function $f : X \to Y$ from a topological space X to a Hausdorff space Y and each f-injective sequence $\{x_n\}_{n\in\omega} \subset X$ that converges to a point $x \in X$ the sequence $\{f(x_n)\}_{n\in\omega}$ converges to the point $f(x)$.

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We say that $f : X \to Y$ is

- *locally finite* at a point $x \in X$ if for some neighborhood $O_x \subset X$ of x, the image $f(O_x)$ is finite;
- *locally infinite* at $x \in X$ if f is not locally finite at x;

Corollary

$X-$ sequentially Hausdorff Fréchet space, $Y-$ Hausdorff space If f is compact-preserving and locally infinite at each point $x \in X$ then $f : X \to Y$ is continuous.

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1. Let $\mathcal{L}$ be a vector space and a set $A \subseteq \mathcal{L}$. We say that A is κ-lineable if $A \cup \{0\}$ contains a κ-dimensional vector space;
2. Let $\mathcal{L}$ be a Banach space and a set $A \subseteq \mathcal{L}$. We say that A is spaceable if $A \cup \{0\}$ contains an infinite dimensional closed vector space;
3. Let $\mathcal{L}$ be a linear commutative algebra and a set $A \subseteq \mathcal{L}$. We say that A is κ-algebrable if $A \cup \{0\}$ contains a κ-generated algebra B (i.e. the minimal system of generators of B has cardinality κ).

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Let $\mathcal{L}$ be a linear commutative algebra and a set $A \subseteq \mathcal{L}$. We say that A is strongly κ-algebrable if $A \cup \{0\}$ contains a κ-generated algebra B that is isomorphic with a free algebra.

$\mathcal{EDF}(\mathbb{R})$ is the set of all nowhere continuous real functions with $|f(\mathbb{R})| < \omega$.

$\mathcal{EDC}(\mathbb{R})$ is the set of all nowhere continuous compact-to-compact functions.

Theorem

The set $\mathcal{EDF}(\mathbb{R})$ is $2^{\mathfrak{c}}$-algebrable but it is not strongly 1-algebrable.

Corollary

The set $\mathcal{EDC}(\mathbb{R})$ is $2^{\mathfrak{c}}$-algebrable.

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Question 2, Hejnice 2012

Is there a function $f \in \mathcal{EDC}(\mathbb{R})$ that has infinitely many values on each interval?

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$X-$ sequentially Hausdorff Fréchet space, $Y-$ Hausdorff space If f is compact-preserving and locally infinite at each point $x \in X$ then $f : X \to Y$ is continuous.

Answer

No.

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$X-$ sequentially Hausdorff Fréchet space, $Y-$ Hausdorff space If f is compact-preserving and locally infinite at each point $x \in X$ then $f : X \to Y$ is continuous.

Answer

No.

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Thank you for your attention :)