wQN_* and wQN^*

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Hejnice

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- functions from X to $\mathbb{R}$
- convergence of sequence $\langle f_n : n \in \omega \rangle$ - pointwise convergence
- quasi-normal convergence of sequence $\langle f_n : n \in \omega \rangle$ if there exists a limit function f and a sequence of positive reals $\langle \varepsilon_n : n \in \omega \rangle$ converging to zero such that

$$|f_n(x) - f(x)| < \varepsilon_n$$

holds for all but finitely many $n \in \omega$, for any $x \in X$

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Sequences

QN-property

X has the property QN if each sequence of continuous functions converging to zero is converging to zero quasi-normally.

wQN-property

X has the property wQN if each sequence of continuous functions converging to zero has a subsequence converging to zero quasi-normally.

SSP-property

X has the property SSP if for each sequence of sequences $\langle\langle f_{n,m} : m \in \omega\rangle : n \in \omega\rangle$ of continuous functions such that $f_{n,m} \to 0$ for any $n \in \omega$, there exists a sequence $\langle m_n : n \in \omega\rangle$ such that $f_{n,m_n} \to 0$.

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Coverings

$S_1(\mathcal{A},\mathcal{B})$-property

Let $\mathcal{A}$, $\mathcal{B}$ be families of covers of a space X. X possesses the property $S_1(\mathcal{A},\mathcal{B})$ if for every sequence $\langle \mathcal{U}_n : n \in \omega \rangle$ of covers from $\mathcal{A}$ there exist sets $U_n \in \mathcal{U}_n$ such that $\{U_n;\ n \in \omega\} \in \mathcal{B}$.

Some covers:

- **γ-cover** $\mathcal{U}$ - every $x \in X$ lies in all but finitely many members of $\mathcal{U}$
  - a family of all countable open γ-covers: $\mathbf{\Gamma(X)}$, $\mathbf{\Gamma}$
- **shrinkable γ-cover** $\mathcal{U}$ - a γ-cover with the property that there exists a closed γ-cover $\mathcal{V}$ which is a refinement of $\mathcal{U}$
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Relations

$$\begin{array}{ccccc} & & & & \mathrm{SSP} \\ & & & & \| \\ \mathrm{QN} & \longrightarrow & S_1(\Gamma,\Gamma) & \longrightarrow & S_1(\Gamma^{sh},\Gamma) \\ & & & & \| \\ & & & & \mathrm{wQN} \end{array}$$

1. There is a model of ZFC with an $S_1(\Gamma,\Gamma)$-set which is not a QN-set.
2. There is a model of ZFC, where all of these properties are equivalent.

Scheepers conjecture

Perfectly normal wQN-space has property $S_1(\Gamma,\Gamma)$.

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A function f is lower semicontinuous if for any real number r the set

$$f^{-1}((r,\infty)) = \{x \in X : f(x) > r\}$$

is **open** in a space X.

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Sequences of semicontinuous functions

Bukovsky L.: *On* wQN_* *and* wQN^* *spaces* (2008):

wQN$_*$ and **SSP**$_*$- as wQN and SSP but continuous functions are substituted by lower semicontinuous ones in their definitions

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What are the direct proofs of implications

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H. Ohta, M. Sakai: *Sequences of semicontinuous functions accompanying continuous functions* (2009):

A space X has property (USC), if whenever $\langle f_n : n \in \omega \rangle$ of **upper semicontinuous functions** with $f(X) \subseteq [0, 1]$ converges to zero, there is $\langle g_n : n \in \omega \rangle$ of **continuous functions** converging to zero such that

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For f, an upper semicontinuous function with $f(X) \subseteq [0, 1]$, there is a sequence $\langle f_n : n \in \omega \rangle$ of **continuous functions** such that $\boldsymbol{f_n \searrow f}$.

Let $\langle f_n : n \in \omega \rangle$ be a sequence of upper semicontinuous functions with $f_n(X) \subseteq [0, 1]$ converging to zero.Then

- there is $\langle f_{n,m} : m \in \omega \rangle$, such that $f_{n,m} \searrow f_n$ for every $n \in \omega$
- $f_{n,m} - f_n \searrow 0$ and $f_{n,m} - f_n$ is lower semicontinuous function
- by SSP$_*$ there is $\varphi \in {}^\omega\omega$ such that $f_{n,\varphi(n)} - f_n \searrow 0$
- $f_{n,\varphi(n)}$ is required sequence for (USC)

- SSP$_* \longrightarrow$ (USC)

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Let $\langle f_n : n \in \omega \rangle$ be a sequence of upper semicontinuous functions with $f_n(X) \subseteq [0,1]$ converging to zero.Then

- there is $\langle f_{n,m} : m \in \omega \rangle$, such that $f_{n,m} \searrow f_n$ for every $n \in \omega$
- $f_{n,m} - f_n \searrow 0$ and $f_{n,m} - f_n$ is lower semicontinuous function
- by SSP_* there is $\varphi \in {}^\omega\omega$ such that $f_{n,\varphi(n)} - f_n \searrow 0$
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- $\mathrm{SSP}_* \rightarrow$ (USC)

Let $\langle\langle f_{n,m} : m \in \omega\rangle : n \in \omega\rangle$ be a sequence of sequences of upper semicontinuous functions with $f(X) \subseteq [0, 1]$ converging to zero. Then

- by (USC) there is $\langle\langle g_{n,m} : m \in \omega\rangle : n \in \omega\rangle$ of continuous functions converging to zero such that $f_{n,m} \le g_{n,m}$
- by SSP_* there is $\varphi \in {}^\omega\omega$ such that $g_{n,\varphi(n)} \to 0$
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- $SSP_* \to SSP^*$

- $wQN_* \to SSP_*$

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Changed range

Let $f : X \to [0, 1]$ be lower semicontinuous function.

- $f^{-1}((r, 1]) = (-f)^{-1}([-1, r))$
- $(-f) : X \to [-1, 0]$ is upper semicontinuous function
- $\mathrm{SSP}^*([-1, 0]) \to \mathrm{SSP}_*([0, 1])$

Properties related to semicontinuous functions depend on ranges of functions taken in their definitions.

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Properties related to semicontinuous functions depend on ranges of functions taken in their definitions.

Variable range

Let $A \in \{\mathbb{R},\ [-1,1],\ [0,\infty),\ [0,1]\}$.

QN(A)- as QN with functions restricted to range A

wQN(A), **wQN$_*$(A)**, **wQN*(A)**- as wQN, wQN$_*$, wQN* with functions restricted to range A

SSP(A), **SSP$_*$(A)**, **SSP*(A)**- as SSP, SSP$_*$, SSP* with functions restricted to range A

QN($\mathbb{R}$) = QN($[-1,1]$) = QN($[0,\infty)$) = QN($[0,1]$) = QN

wQN($\mathbb{R}$) = wQN($[-1,1]$) = wQN($[0,\infty)$) = wQN($[0,1]$) = wQN

SSP($\mathbb{R}$) = SSP($[-1,1]$) = SSP($[0,\infty)$) = SSP($[0,1]$) = SSP

Variable range

Let $A \in \{\mathbb{R},\ [-1,1],\ [0,\infty),\ [0,1]\}$.

QN(*A*)- as QN with functions restricted to range A

wQN(*A*), **wQN$_*$(*A*)**, **wQN*(*A*)**- as wQN, wQN$_*$, wQN* with functions restricted to range A

SSP(*A*), **SSP$_*$(*A*)**, **SSP*(*A*)**- as SSP, SSP$_*$, SSP* with functions restricted to range A

QN($\mathbb{R}$) = QN($[-1,1]$) = QN($[0,\infty)$) = QN($[0,1]$) = QN

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$\mathrm{QN}(\mathbb{R}) \equiv \mathrm{QN}([-1,1]) \equiv \mathrm{QN}([0,\infty)) \equiv \mathrm{QN}([0,1]) \equiv$ **QN**

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QN(A)- as QN with functions restricted to range A

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Let $A \in \{\mathbb{R},\ [-1,1],\ [0,\infty),\ [0,1]\}$.

QN(A)- as QN with functions restricted to range A

wQN(A), **wQN$_*$(A)**, **wQN*(A)**- as wQN, wQN$_*$, wQN* with functions restricted to range A

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Better diagram

$$
\begin{array}{ccccc}
\mathsf{SSP}_*([0,1]) & \longrightarrow & \mathsf{SSP}^*([0,1]) & \longrightarrow & \mathsf{SSP} \\
\| & & \| & & \| \\
\mathsf{QN} & \longrightarrow & \mathsf{S}_1(\Gamma,\Gamma) & \longrightarrow & \mathsf{S}_1(\Gamma^{\mathsf{sh}},\Gamma) \\
\| & & \| & & \| \\
\mathsf{wQN}_*([0,1]) & \longrightarrow & \mathsf{wQN}^*([0,1]) & \longrightarrow & \mathsf{wQN}
\end{array}
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What are the relations among $\mathrm{wQN}_*([-1,1])$, $\mathrm{SSP}_*([-1,1])$, $\dots$?

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Equivalent properties

$\mathrm{wQN}^*([0,1])$	$\mathrm{SSP}^*([0,1])$
$\mathrm{wQN}^*([0,\infty))$	$\mathrm{SSP}^*([0,\infty))$

$\mathrm{wQN}_*([0,1])$	$\mathrm{SSP}_*([0,1])$
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$\mathrm{wQN}_*([-1,1])$	$\mathrm{SSP}_*([-1,1])$
$\mathrm{wQN}^*([-1,1])$	$\mathrm{SSP}^*([-1,1])$
$\mathrm{wQN}_*(\mathbb{R})$	$\mathrm{SSP}_*(\mathbb{R})$
$\mathrm{wQN}^*(\mathbb{R})$	$\mathrm{SSP}^*(\mathbb{R})$

Equivalent properties

$wQN^*([0,1])$	$SSP^*([0,1])$
$wQN^*([0,\infty))$	$SSP^*([0,\infty))$

$wQN_*([0,1])$	$SSP_*([0,1])$
$wQN_*([0,\infty))$	$SSP_*([0,\infty))$
$wQN_*([-1,1])$	$SSP_*([-1,1])$
$wQN^*([-1,1])$	$SSP^*([-1,1])$
$wQN_*(\mathbb{R})$	$SSP_*(\mathbb{R})$
$wQN^*(\mathbb{R})$	$SSP^*(\mathbb{R})$

Equivalent properties

$\mathrm{wQN}^*([0,1])$	$\mathrm{SSP}^*([0,1])$
$\mathrm{wQN}^*([0,\infty))$	$\mathrm{SSP}^*([0,\infty))$

$\mathrm{wQN}_*([0,1])$	$\mathrm{SSP}_*([0,1])$
$\mathrm{wQN}_*([0,\infty))$	$\mathrm{SSP}_*([0,\infty))$
$\mathrm{wQN}_*([-1,1])$	$\mathrm{SSP}_*([-1,1])$
$\mathrm{wQN}^*([-1,1])$	$\mathrm{SSP}^*([-1,1])$
$\mathrm{wQN}_*(\mathbb{R})$	$\mathrm{SSP}_*(\mathbb{R})$
$\mathrm{wQN}^*(\mathbb{R})$	$\mathrm{SSP}^*(\mathbb{R})$

Thanks for your attention!