Randomness and Universality inTopological Spaces

Rachel Zachariah
Dissertation Supervisor:Dr.Rolf Suabedissen

Department of Mathematics
University Oxford

Jun 18, 2012

Outline

Outline

The Erdös-Renyii Theorem

- The random graph on countably many vertices is universal with probability one.
  [*P. Erdös, A. Rényi. Asymmetric graphs. Acta Math. Acad. Sci. Hungar. 14 (1963)*]

Outline

Main result of the paper:

- The random Polish Space is almost surely the Universal Urysohn Space.
[*A.M.Vershik, 2004*]

Outline

On the Urysohn space

- We consider Polish spaces as objects of a different category than metric spaces and isometries, for example, the category of `uniform spaces` and `uniform isomorphisms` or the category of `topological spaces` and `homeomorphisms`.

On the Urysohn space

- Every bi-Lipschitz function between finite subsets of the Urysohn space can be extended to a bi-Lipschitz function from the whole space onto itself.
(follows from a theorem of W.Kubis, M.Rubin in *Extension and reconstruction theorems for the Urysohn universal metric space, 2008*)
- If we consider the sub-category of `Polish spaces` and `uniform isomorphisms`, the Universal Urysohn space satisfies universality and ω-homogeneity conditions.
- Similarly in the sub-category of `Polish spaces` and `homeomorphisms`, the Universal Urysohn space satisfies universality and ω-homogeneity conditions.

Outline

Other ways to generate random spaces

- *Initial Topologies*:
  - The random space generated this way is almost surely homeomorphic to the rationals.
  - Every second countable,regular,countable space is homeomorphic to a subspace of the rationals $\mathbb{Q}$.
    And $\mathbb{Q}$ is the unique (upto homeomorphism) space with this property that is strongly ω-homogeneous.
- *Random Closure Operators*: The random space almost always has the indiscrete topology.