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$István Juhász (Rényi Institute)$
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QUESTION. Are MN spaces maximally resolvable?
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**PROBLEM.** (Ceder and Pearson, 1967)
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Main results of [J-S-Sz]

– If $\kappa$ is measurable then there is a MN space $X$ with $\Delta(X) = \kappa$ that is not $\omega_1$-resolvable.
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This left a number of questions open.
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– [Kunen - Prikry, 1971] If \( \lambda < \aleph_\omega \) then every \( \mathcal{F} \in \text{un}(\lambda) \) is maximally decomposable.

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**FACT.** [J-S-Sz] Every filtration space $X(F)$ is MN: For $s \in V \in \tau_F$ put

$$ H(s, V) = \{ t \in V : s \leq t \text{ and } [s, t] \subset V \} $$
irresolvability of ultrafiltration spaces
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COROLLARY. [J-S-Sz]

If $\mathcal{F} \in \text{un}(\kappa)$ is a measure and $F(t) = \mathcal{F}$ for all $t \in \text{dom}(F) = \kappa^{<\omega}$ then $X(F)$ is hereditarily $\omega_1$-irresolvable.
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Full $\lambda$-filtrations were considered in [J-S-Sz].
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THEOREM [J-M]

For $\lambda$ singular and $\text{cf}(\lambda)^+ < \kappa \leq \lambda$, TFAEV

– Every DSD space $X$ with $|X| = \Delta(X) = \lambda$ is $\kappa$-resolvable.
– Every MN space $X$ with $|X| = \Delta(X) = \lambda$ is $\kappa$-resolvable.
– $X(F)$ is $\kappa$-resolvable for every $\lambda$-filtration $F$.

NOTE. In both results, the case $\kappa = \lambda$ is of main interest.
idea of proof
Lemma. [J-S-Sz]
If every $x \in X$ is the complete accumulation point of a SD set $Y \subset X$ with $|Y| = \lambda$ then there is a full $\lambda$-filtration $F$ and a one-one continuous map $g : X(F) \to X$. 
Lemma. [J-S-Sz]

If every \( x \in X \) is the complete accumulation point of a SD set \( Y \subset X \) with \( |Y| = \lambda \) then there is a full \( \lambda \)-filtration \( F \) and a one-one continuous map \( g : X(F) \to X \).

Assume that \( \lambda \) is regular, \( X \) is DSD with \( |X| = \Delta(X) = \lambda \), and \( x \in X \) is not a complete accumulation point of any SD set \( Y \in [X]^{\lambda} \).
Lemma. [J-S-Sz]

If every \( x \in X \) is the complete accumulation point of a SD set \( Y \subset X \) with \( |Y| = \lambda \) then there is a full \( \lambda \)-filtration \( F \) and a one-one continuous map \( g : X(F) \to X \).

Assume that \( \lambda \) is regular, \( X \) is DSD with \( |X| = \Delta(X) = \lambda \), and \( x \in X \) is not a complete accumulation point of any SD set \( Y \in [X]^\lambda \). Then \( x \in T_\lambda(X) \).
Lemma. [J-S-Sz]

If every $x \in X$ is the complete accumulation point of a SD set $Y \subset X$ with $|Y| = \lambda$ then there is a full $\lambda$-filtration $F$ and a one-one continuous map $g : X(F) \to X$.

Assume that $\lambda$ is regular, $X$ is DSD with $|X| = \Delta(X) = \lambda$, and $x \in X$ is not a complete accumulation point of any SD set $Y \in [X]^\lambda$. Then $x \in T_\lambda(X)$. But if $T_\lambda(X)$ is dense in $X$, then $X$ is $\lambda$-resolvable.
Lemma. [J-S-Sz]

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Assume that \( \lambda \) is regular, \( X \) is DSD with \( |X| = \Delta(X) = \lambda \), and \( x \in X \) is not a complete accumulation point of any SD set \( Y \in [X]^\lambda \). Then \( x \in T_\lambda(X) \). But if \( T_\lambda(X) \) is dense in \( X \), then \( X \) is \( \lambda \)-resolvable.

This takes care of the case when \( \lambda \) is regular.
Lemma. [J-S-Sz]

If every $x \in X$ is the complete accumulation point of a SD set $Y \subset X$ with $|Y| = \lambda$ then there is a full $\lambda$-filtration $F$ and a one-one continuous map $g : X(F) \to X$.

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This takes care of the case when $\lambda$ is regular.

The singular case (proved in [J-M]) is similar but more complicated.
\( \lambda \)-resolvability of \( \lambda \)-filtration spaces

István Juhász (Rényi Institute)
THEOREM [J-M]

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If $\kappa \leq \lambda$ and $F$ is a $\lambda$-filtration s.t.
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(i) for every $t \in T = \text{dom}(F)$, if $\mu_t \geq \kappa$ then $F(t)$ is $\kappa$-decomposable,
THEOREM [J-M]

If $\kappa \leq \lambda$ and $F$ is a $\lambda$-filtration s.t.

(i) for every $t \in T = \text{dom}(F)$, if $\mu_t \geq \kappa$ then $F(t)$ is $\kappa$-decomposable,

(ii) for every $t \in T = \text{dom}(F)$ and $\mu \leq \kappa$,

$$\{ \alpha < \mu_t : F(t \cap \alpha) \text{ is } \mu\text{-decomposable} \} \in F(t) ,$$
\( \lambda \)-resolvability of \( \lambda \)-filtration spaces

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\]

then \( X(F) \) is \( \kappa \)-resolvable.
\( \lambda \)-resolvability of \( \lambda \)-filtration spaces

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**COROLLARY [J-M]**

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resolvable

Hejnice 2012

12 / 12
\( \lambda \)-resolvability of \( \lambda \)-filtration spaces

**THEOREM [J-M]**

If \( \kappa \leq \lambda \) and \( F \) is a \( \lambda \)-filtration s.t.

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\]

then \( X(F) \) is \( \kappa \)-resolvable.

**COROLLARY [J-M]**

If every \( F \in \text{un}(\mu) \) is maximally decomposable whenever \( \omega \leq \mu \leq \lambda \),

then \( X(F) \) is \( \lambda \)-resolvable for any \( \lambda \)-filtration \( F \).