Algebraic characterisation of pseudo-elementary and second-order classes

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In this talk I will present purely algebraic (model-theoretic) characterisations for classes definable in second-order logic and for pseudo-elementary classes (including PC and PC_Δ classes). Classical results of this flavour include Keisler–Shelah type theorems (characterising first-order definability by closure under ultraproducts and ultraroots) and Birkhoff's **HSP** theorem; a key starting point for this talk is Sági's work [1], which provides an algebraic description of classes definable by existential second-order sentences. Here we resolve several open problems from [2] and [1].

Our main results are the following.

- We solve the long-standing problem of giving a purely algebraic characterisation of pseudo-elementary classes: we characterise PC_Δ classes by intrinsic closure properties. We also give a characterisation for the basic pseudo-elementary classes (PC), and, using this description we obtain an internal (closure-based) criterion for when a class of finite structures lies in NP.
- We provide a structural classification of second-order equivalent structures, and we obtain purely algebraic characterisations of the classes definable by second-order formulas as well as those definable by finitely many second-order sentences.

Technically, our approach blends ultraproduct methods with the topological analysis of certain subspaces of 2^κ. With these tools we aim to provide a new perspective on the understanding of axiomatisability phenomena.

References

[1] G. Sági, *Ultraproducts and Higher Order Formulas,* Math. Logic Quarterly, Vol. 48, No. 2, pp. 261–275, (2002).

[2] Keisler, H. Jerome. *Limit ultraproducts.* The Journal of Symbolic Logic 30.2 (1965): 212-234.