Structural Infinite-Exponent Partition Relations and Weak Choice Principles

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It has been known since at least [ER52] that infinite-exponent partition relations (IEPRs) are inconsistent with AC, i.e. that no higher analogue of Ramsey's theorem can hold for colourings of infinite tuples; in ZF without AC, however, such relations can hold. The same is true of *structural* IEPRs, i.e. ones in which the "tuples" being coloured are the isomorphic copies of some given structure inside a larger structure (see [Ga25]). In this talk we will discuss some particular structural IEPRs which are consistent with ZF but which imply failures of fragments of Choice such as the Kinna-Wagner selection principle KWP_1 and the Ordering Principle O.

References

[ER52] P. Erdős, R. Rado, *Combinatorial theorems on classifications of subsets of a given set*, Proceedings of the London Mathematical Society, Volume s3–2 (1952), pp. 417–439

[Ga25] L. A. Gardiner, *Infinite-exponent partition relations on the real line*, arXiv:2507.12361 (2025)