Baire Category Invariants and the Structure of Non-Separable Banach Spaces

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The presentation will be a survey of results from [1], studied as part of my Master's research. Therefore, the theorems discussed below are due to the author of the cited paper.

One of the most effective ways to analyze and interpret a Banach space is via Schauder bases. Although every vector space has an algebraic basis, this is not always true for bases of this sort. Long Schauder bases arise as an attempt to define a similar notion for non-separable spaces.

Definition 1. *A transfinite sequence of vectors $\{x_\gamma\}_{\gamma<\Gamma} \subseteq X$ is called a **long Schauder basis** if, for every $x \in X$, there exists a unique transfinite sequence of scalars $(\alpha_\gamma)_{\gamma<\Gamma}$ such that*

$$x = \sum_{\gamma<\Gamma} \alpha_\gamma x_\gamma.$$

Since not every Banach space possesses a long Schauder basis, it is often necessary to restrict ourselves to closed subspaces or quotients in order to utilize such structures. It is natural, then, to ask whether every infinite-dimensional Banach space possesses an infinite-dimensional quotient with a Schauder basis.

In the work investigated here [1], Todorcevic utilizes Baire category methods to demonstrate that the answer is positive provided that the density of the space is bounded by a specific cardinal invariant $\mathfrak{mm}$. The main result is stated as follows:

Theorem 1 (Theorem 5 in [1])**.** *Every Banach space X of density $< \mathfrak{mm}$ has a quotient with a monotone Schauder basis which could be chosen to have uncountable length if X is nonseparable.*

References

[1] TODORCEVIC, S. Biorthogonal systems and quotient spaces via Baire category methods. **Mathematische Annalen**, Berlin, 335, 687–715, 2006.

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