Baire Category Invariants and the Structure of Non-Separable Banach Spaces

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Keywords: Banach spaces, biorthogonal systems, Martin's maximum, long schauder basis.

This presentation serves as a survey of results based on [1], studied within the scope of a Master's degree. It is emphasized that the theorems presented are due to the author of the cited work.

One of the most effective ways to analyze and interpret a Banach space is through Schauder bases. Unlike algebraic bases, which rely on finite linear combinations, Schauder bases allow for infinite expansion. Classically, a Schauder basis is defined as a countable sequence $\{e_n\}_{n\in\mathbb{N}}$, which restricts its existence to separable Banach spaces. To analyze non-separable spaces, this concept is extended to Long Schauder bases, which are indexed by transfinite ordinals.

Definition 1. *A transfinite sequence of vectors $\{x_\gamma\}_{\gamma<\Gamma} \subseteq X$ is called a **long Schauder basis** if, for every $x \in X$, there exists a unique transfinite sequence of scalars $(\alpha_\gamma)_{\gamma<\Gamma}$ such that*

$$x = \sum_{\gamma<\Gamma} \alpha_\gamma x_\gamma,$$

where the sum converges in the norm topology.

Since not every Banach space possesses a long Schauder basis, it is often necessary to work with closed subspaces or quotients. It is natural, then, to ask whether every non-separable Banach space possesses a quotient with a long Schauder basis.

In the work investigated here [1], Todorcevic utilizes Baire category methods to demonstrate that the answer is positive provided that the density of the space is bounded by a specific cardinal invariant $\mathfrak{mm}$. The main result is stated as follows:

Theorem 1 (Theorem 5 in [1])**.** *Every Banach space X of density $< \mathfrak{mm}$ has a quotient with a monotone Schauder basis. Furthermore, if X is nonseparable, this basis can be chosen to have uncountable length.*

References

[1] TODORCEVIC, S. Biorthogonal systems and quotient spaces via Baire category methods. **Mathematische Annalen**, Berlin, 335, 687–715, 2006.

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