Topological Stäckel Hypothesis

Dominik Bargieła

January 2026

There are many different notions of compactness, often obtained by generalizing properties of compact (especially finite) spaces. For example, every continuous real-valued function on a compact space is bounded; this generalizes to spaces called *pseudocompact*, i.e. a topological space X is *pseudocompact* if every continuous real-valued function on X is bounded.

Two years ago, a new type of compactness was introduced by Abhijit Dasgupta (see [1]). It is known that in a compact ordered space, every non-empty closed subset has both a minimal and a maximal element. This motivates the following definition: a topological space is *Stäckel-compact* if it admits a linear ordering such that every non-empty closed subset has both a minimal and a maximal element. The name of this notion is inspired by the theorem of Paul Stäckel (see [2]), which characterizes finite sets in terms of this property.

I will present several results on Stäckel-compact spaces and introduce the central open problem concerning them, which I call the *Topological Stäckel Hypothesis.* A positive answer to this question would provide a surprising characterization of countable compactness for Hausdorff spaces.

References

[1] A. Dasgupta, *Compactness and symmetric well-orders,* Bulletin of the Polish Acad. Sci. Math., 2024.

[2] P. Stäckel, *Zu H. Webers Elementarer Mengenlehre,* Jahresber, d.d. M.-V. 16, 1907.