On the structure of commutative, (von Neumann) regular, semiartinian algebras

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The structure of semiartinian (von Neumann) regular ring with primitive factors artinian is captured by invariant called dimension sequence D_R (described in [1]), which consists of the various skew-fields and dimensions occurring in the layers of the socle sequence of R. Though D_R does not determine R up to an isomorphism even for rings of Loewy length 2, we prove constructively (in [2]) that it does so when R is a commutative semiartinian regular K-algebra of countable type over a field K. Moreover, we prove that the K-algebras coming from the same extended construction (which are subalgebras of K^κ for some cardinal κ) possess conormed strong multiplicative bases despite the fact that the ambient K-algebras K^κ do not even have any bounded bases for any infinite cardinal κ.

In the talk, this joint work with Jan Trlifaj will be overviewed as a generalization of some old results about superatomic Boolean algebras.

[1] P. Růžička, J. Trlifaj, J. Žemlička, Criteria of Steadiness, Marcel Dekker, Abelian Groups, Module Theory and Topology, 1998

[2] K. Fuková, J. Trlifaj, Multiplicative bases and commutative semiartinian von Neumann regular algebras, arXiv:2501.06018, 2025