IDEAL ZOO IN THE BAIRE SPACE 1

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The results were obtained in collaboration with M. Michalski and Sz. Żeberski and are written down in [3].

The idea comes from the following characterization, which can be found in [1, Lemma 2.5.1]:

Lemma 1. *Suppose that $F \subseteq 2^\omega$ has measure zero. Then there exists a sequence $(S_n)_{n \in \omega}$, $S_n \subseteq 2^n$, such that $\sum_{n \in \omega} \frac{S_n}{2^n} < \infty$ and*

$$F \subseteq \{x \in 2^\omega : (\exists^\infty n \in \omega)(x {\restriction} n \in S_n)\}.$$

In [2] we rewrite the above characterization directly in a context of the Baire space to obtain a natural example of an ideal resembling measure. However, the condition that limits the size of S_n s does not seem natural in the Baire space. Therefore, we would like to parametrize it with a function $h \in \omega^\omega$, introducing "fake-null" sets.

Definition 2. *Let $h \in \omega^\omega$, $\limsup_n h(n) = \infty$. We say that $F \in f\mathcal{N}(h)$ if there is a sequence $(S_n)_{n \in \omega}$, $S_n \subseteq \omega^n$, $\sum \frac{|S_n|}{h(n)} < \infty$, satisfying a condition*

$$F \subseteq \{x \in \omega^\omega : (\exists^\infty n)(x {\restriction} n \in S_n)\}.$$

We will also define fake-small and fake-$\mathcal{E}$ sets. We will explore properties of those objects (in particular being and ideal) and investigate their cardinal invariants (where it makes sense).

Moreover, we will also talk about a σ-ideal $\mathcal{M}_-$, which comes from the rewriting of the known characterization of the meager sets into the Baire space.

Definition 3. *$F \in \mathcal{M}_-$ if there are $x_F \in \omega^\omega$ and a partition of ω into intervals $(I_n)_{n \in \omega}$ such that*

$$F \subseteq \{x \in \omega^\omega : (\forall^\infty n)(x {\restriction} I_n \neq x_F {\restriction} I_n)\}.$$

REFERENCES

[1] T. Bartoszyński and H. Judah. *Set theory: On the structure of the real line*. A K Peters. Ltd., 1995.

[2] Łukasz Mazurkiewicz, Marcin Michalski, Robert Rałowski, and Szymon Żeberski. "On algebraic sums, trees and ideals in the Baire space". In: *arXiv:2409.17748* (2024).

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