A 2 generator free LD-algebra of embeddings

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A set with a binary operation $*$ is called an *LD-algebra* if it satisfies

$$a * (b * c) = (a * b) * (a * c);$$

that is, if left multiplication by any given element a is a homomorphism of the structure. Now, the large cardinal axiom I3, which lies with its variants at the very top of the large cardinal hierarchy (at least, the part compatible with Choice), posits the existence of *rank-to-rank embeddings* — non-trivial elementary embeddings from some V_λ to itself. Given a λ for which such embeddings exist, there is a natural binary operation of "application" on these embeddings which is easily seen to give an LD-algebra; Laver showed that in fact, the algebra generated by a single such embedding under application is the free LD-algebra on 1 generator. Many other rank-to-rank embeddings must also exist whenever at least one does, so a natural question is whether Laver's result can be pushed further: does the existence of a rank-to-rank embedding give rise to a free 2-generated LD-algebra of embeddings? I will present joint work with Scott Cramer and Sheila Miller Edwards, showing that from a slightly stronger large cardinal assumption (but one still compatible with Choice, namely, I2) we indeed get a free 2-generated LD-algebra of embeddings. Finally, with a couple of insights from Gabe Goldberg, our argument can be modified to only need the original assumption I3.