

In the talk we focus on the relation of countable tightness of the space $P(K)$ of Radon probability measures on a compact Hausdorff space K and of existence of measures in $P(K)$ that have uncountable Maharam type. Recall that a topological space X has countable tightness if any element of the closure of a subset A of X lies in the closure of some countable subset of A . A Maharam type of a Radon probability measure μ is the density of the Banach space $L_1(\mu)$.

It was proven by Fremlin that, under Martin's axiom and negation of continuum hypothesis, for a compact Hausdorff space K the existence of a Radon probability of uncountable type is equivalent to the existence of a continuous surjection from K onto $[0, 1]^{\omega_1}$. Hence, under such assumptions, countable tightness of $P(K)$ implies that there is no Radon probability on K which has uncountable type. Later, Plebanek and Sobota showed that, without any additional set-theoretic assumptions, countable tightness of $P(K \times K)$ implies that there is no Radon probability on K which has uncountable type. It is thus natural to ask whether the implication " $P(K)$ has countable tightness \Rightarrow every Radon probability on K has countable type" holds in ZFC.

I will present our joint result with Piotr Koszmider that under diamond principle the answer is negative.

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