Universally meager sets in the Miller model and similar ones

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A space $X \subseteq 2^\omega$ is universally meager if for any Polish space Y and any continuous nowhere constant map $f : Y \to 2^\omega$ the preimage $f^{-1}[X]$ is meager in Y. We present a forcing property (†), which is a strenthening of properness and implies that no dominating reals are added. It is known that many classical forcing posets like Cohen, Sacks and Miller satisfy this property. We showed that property (†) is preserved by countable support iterations.

We used this preservation result to prove that if we have such an iteration of length ω_2 over a model of CH, where the single forcings have size at most ω_1, all universally meager sets $X \subseteq 2^\omega$ have size at most ω_1 in the forcing extension.
This has multiple set-theoretic applications:

- There is no perfectly meager space of size continuum in the Miller model.
- There is no strong measure zero set of size continuum in the Miller model.
- Miller proved in 2005 that there exists a strong measure zero set of size ω_1 iff there exists a Rothberger space of size ω_1. We get that it is consistent with ZFC to have a strong measure zero set of size ω_2, but no Rothberger spaces of size ω_2.

This is joint work with Piotr Szewczak (Cardinal Stefan Wyszyński University in Warsaw) and Lyubomyr Zdomskyy (TU Vienna).