Laver Ultrafilters

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We call an ultrafilter $\mathcal{U}$ a *Laver ultrafilter* if the relativized Laver forcing $\mathbb{L}_{\mathcal{U}}$ has the Laver property. In this talk, we give a simple combinatorial characterisation of Laver ultrafilters, and further characterize them as $\mathcal{I}$-ultrafilters – in the sense of Baumgartner [1] – for a certain class of ideals $\mathcal{I}$ on 2^ω.

From these characterisations it follows that the class of Laver ultrafilters (properly) contains the rapid P-points, is (properly) contained in the class of hereditarily rapid ultrafilters, and is closed under sums. While it follows from a result due to Błaszczyk and Shelah [4] that Laver ultrafilters are necessarily nowhere dense, they actually have the stronger property of being measure zero. In fact, they are $\mathcal{I}_f$-ultrafilters for each so-called *Yorioka ideal* $\mathcal{I}_f$, approximations of the strong measure zero ideal.

On the other hand, the existence of a non-scattered Laver ultrafilter under MA shows that – among the ideals $\mathcal{I}$ on 2^ω considered by Baumgartner [1], Barney [2], and Brendle [3] – the above inclusions between the classes of Laver- and $\mathcal{I}$-ultrafilters are the only ones provable in ZFC.

Finally, we will talk about the existence of Laver ultrafilters in models without P-points.

This is work in progress with Tan Özalp.

References

[1] Baumgartner, James, **Ultrafilters on** ω, The Journal of Symbolic Logic, 60, 2, pages 624–639, 1995, Cambridge University Press.

[2] Barney, Christopher **Ultrafilters on the natural numbers**, The Journal of Symbolic Logic, 68, 3, pages 764–784, 2003, Cambridge University Press

[3] Brendle, Jörg **Between P-points and nowhere dense ultrafilters**, Israel Journal of Mathematics, 113, 1, pages 205–230, 1999, Springer

[4] Błaszczyk, Aleksander and Shelah, Saharon **Regular subalgebras of complete Boolean algebras**, The Journal of Symbolic Logic, 66, 2, pages 792–800, 2001, Cambridge University Press