THE HIGHER CLOSED NULL IDEAL(S)

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ABSTRACT. The *closed null ideal* $\mathcal{E}$ is the σ-ideal on the Cantor space ${}^{\omega}2$ generated by subsets that are simultaneously topologically closed and of Lebesgue measure zero. It is well-known that every closed null set is meagre, thus $\mathcal{E}$ is a subset of the meagre ideal $\mathcal{M}$.

The *higher Cantor space* is the set of functions ${}^{\kappa}2$, where κ is regular uncountable and $\kappa = \kappa^{<\kappa}$, generated by the $<\kappa$-box topology. Agostini, Barrera & Dimonte (arXiv:2601.13321) have very recently put the final nail in the coffin of any attempt to define a suitable notion of Lebesgue measure on ${}^{\kappa}2$. Nevertheless, we will show that it is possible to generalise the *closed null ideal*, using a combinatorial characterisation of $\mathcal{E}$. In fact, we will define three distinct $\leq\kappa$-complete ideals, $\mathcal{H}_\kappa$, $\mathcal{E}_\kappa$ and $\mathcal{BE}_\kappa$, each contained in the κ-meagre ideal $\mathcal{M}_\kappa$, and we will argue that each could be considered the higher analogue of $\mathcal{E}$.

If time permits, we will compare cardinal functions (the additivity, uniformity, covering and cofinality numbers) of these ideals to other higher cardinal characteristics, such as the bounding, dominating and splitting numbers.

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