The Closed Subtree Property for Aronszajn trees

Fumiaki Nishitani[*]
Shizuoka University, Japan

Justin T. Moore in 2005 introduced the closed subtree property (csp) for Aronszajn trees. Also, Moore showed that if $\mathsf{MA}_{\aleph_1}$ holds, then every Aronszajn tree has csp. This property can be seen as a generalization of the club-embeddability into all subtrees of an Aronszajn tree in terms of the negation of the weak diamond principle (see [1]).

In this talk, we construct Aronszajn trees with/without csp by using the diamond principle. Furthermore, we show that Moore's result can be extended by proving his forcing notion has stronger properties than the countable chain condition. Finally, we introduced a variation of the eventual precaliber $\aleph_1$, which was studied by Aoki (see [2]), and show that any forcing having this property preserves a subtree with no uncountable closed subtrees.

References

[1] J. T. Moore. ω_1 and $-\omega_1$ may be the only minimal uncountable linear orders. *Michigan Math. J.*, Vol. 55, No. 2, pp. 437–457, 2007.

[2] Yushiro Aoki. Discontinuous homomorphisms on $C(X)$ with the negation of CH and a weak forcing axiom. *J. Lond. Math. Soc. (2)*, Vol. 110, No. 1, pp. Paper No. e12956, 39, 2024.

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[*]nishitani.fumiaki.19@shizuoka.ac.jp