On uniformly Menger elements

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A subset A of a space X is relatively Menger if for every sequence $\{C_n : n \in \mathbb{N}\}$ of open covers of X, there exists, for each n a finite set $J_n \subseteq C_n$ such that $A \subseteq \bigcup_{n\in\mathbb{N}} J_n$. From the notion of relatively Menger subsets, we define and study uniformly Menger elements in the category of nearness frames. An element a of a nearness frame (L, μ) is uniformly Menger if for every sequence $\{C_n : n \in \mathbb{N}\}$ of uniform covers of L, there exists, for each n a finite set $D_n \subseteq C_n$ such that $a \leq \bigcup_{n\in\mathbb{N}} D_n$. These elements are situated between totally bounded elements and pre-Lindelöf elements. The study of uniformly Menger elements leads to a consideration of a notion of boundedness in pointfree topology which is in terms of uniform covers. We prove that an element a of a nearness frame (L, μ) is uniformly Menger if and only if the quotient $j_{\downarrow a} : (L, \mu) \to (\downarrow a, \mu_{\downarrow a})$ is uniformly M-bounded.