On the binary linear ordering

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Abstract

We consider the rarefied class of linear order types which are untranscendable (that is to say, they cannot be embedded in a product of two smaller ones) yet divisible (which means that any ordering of said type can be partitioned into two parts such that neither of the induced suborderings embeds the whole ordering). It is a corollary of folklore that there is just one ordinal in this class.

Hausdorff initiated the study of the class of scattered linear orderings—those which do not contain a copy of the rational numbers. Often times properties of the ordinals carry over to this class. We are going to see that this is also the case here—there is but one scattered divisible untranscendable linear ordering.

Another example in the class of divisible untranscendable linear orderings is the real number line. However it belonging to this class hinges on the Axiom of Choice. Towards the end we are going to confront some open problems. This is joint work in progress with Garrett Ervin and Alberto Marcone.

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